Homework #3

due Thursday, October 11, in class

Instructions: Complete all problems and turn in a set of answers either to me or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write "According to Excel, ..." in your answer.

Problem 1 The manager of an automobile dealership is considering a new bonus plan designed to increase sales volume. Currently, the mean sales volume is 14 automobiles per month. The manager wants to conduct a reasearch study to see whether the new bonus plan increases sales volume. To collect data on the plan, a sample of sales personnel will be allowed to sell under the new bonus plan for a one-month period.

a. Develop the null and alternative hypotheses that are most appropriate for this research situation (hint: be careful! Read pages 391-392. Is a one- or two-tailed test more appropriate?)

b. Suppose a limited test is conducted among 25 randomly chosen salespeople, who sell an average of 14.7 cars under the bonus plan. Supopse the standard deviation for cars sold is 3. Test the hypotheses you identify in part a. with $\alpha = .1$, and interpret your conclusion.

Problem 2 The label on a three-quart container of orange juice claims that the orange juice contains an average of one gram of fat or less. Answer the following question for a hypothesis test that could be used to test the claim on the label.

a. Develop appropriate null and alternative hypotheses.

b. What is a Type I error in this situation? What are the consequences of making this error?

c. What is a Type II error in this situation? What are the consequences of making this error?

Problem 3 Individuals filing 1994 federal income tax returns prior to March 31, 2009 had an average refund of \$1,056 (*USA Today*, April 5, 2009). Consider the population of "last-minute" filers who mail their returns during the last five days of the income tax period (typically April 11 through April 15).

a. A researcher suggests that one of the reasons individuals wait until the last five days to file their returns is that on average those individuals have a lower refund than early filers. Develop appropriate hypotheses such that rejection of H_0 will support the researcher's suggestion.

b. For a sample of 400 individuals who filed a return between April 11 and April 15, the sample mean refund was \$910, and the standard deviation is \$1,600. At $\alpha = .05$, what is your conclusion?

c. What is the p-value for this test?

Problem 4 Fightmaster and Associates Real Estate, Inc. advertises that the mean selling time of a residential home is 40 days or less. A sample of 50 recently sold residential homes shows a sample mean selling time of 34 days and a standard deviation of 20 days. Using $\alpha = .02$, test the validity of the company's claim.

Problem 5 A consumer research group is interested in testing an automobile manufacturer's claim that a new sedan will travel at least 35 miles per gallon of gasoline ($H_0 : \mu \geq 35$).

a. With $\alpha = .02$, a sample of 30 cars, and $\sigma = 3mpg$, what is the rejection rule based on the value of $\overline{x}$ for the test to determine whether the manufacturer's claim should be rejected?

b. What is the probability of committing a Type II error if the actual mileage is 33mpg?

c. What is the probability of committing a Type II error if the actual mileage is 34mpg?

d. What is the probability of committing a Type II error if the actual mileage is 34.5mpg?

Problem 6 Consider the following hypotheses:

$$H_0 : \quad \mu = 20$$
$$H_1 : \quad \mu \neq 20$$

a. Determine the p-value of the test when $\overline{x} = 21$, $n = 25$, and $\sigma = 5$:

b. Repeat part a with $\overline{x} = 22$.

c. Repeat part a with $\overline{x} = 23$.

d. Describe what happens to the value of the test statistic and the p-value when the value of $\overline{x}$ increases.

Problem 7 Consider the following hypotheses:

$$H_0 : \quad \mu = 200$$
$$H_1 : \quad \mu \neq 200$$

a. Suppose $\alpha = .05$, $\sigma = 10$, and $n = 100$. What is the probability of type II error if $\mu = 203$?

b. Repeat part a. for $\mu = 205$, $\mu = 207$, and $\mu = 209$

c. Use the information from parts a-b to sketch the operating characteristic curve for this test (refer to pages 390-391).

Problem 8 The following question asks you to perform a hypothesis test about a population proportion, p. In this problem, let p be the fraction of all drivers who wear seat belts. You are interested in the following hypotheses:

$$H_0 : \quad p = .6$$
$$H_1 : \quad p > .6$$

a. Suppose you observe $n = 100$ randomly chosen drivers, and notice that 63 of them are wearing seat belts. Refer to page 422 Keller. The test statistic for hypothesis tests about p is $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$, where p is the population proportion (hypothesized under H_0 to be .6), and $\hat{p}$ is the sample proportion. Just like when performing a hypothesis test on a population mean, the test statistic has a standard normal distribution. Calculate the p-value of the test.

b. Repeat part a with $n = 200$.

c. Repeat part a with $n = 400$.

d. Describe the effect on the p-value of increasing the sample size.

Problem 9 A researcher claims that a recent decrease in the number of airline passengers has resulted in better on-time performance. Before the increase in passengers, one airline bragged that 92% of its flights were on time. A random sample of 165 flights completed this year reveals that 153 were on time. Can we conclude with $\alpha = .05$ that the airline's on-time performance has improved? (Hint: this is another question about proportions. Refer to problem 8.)