Homework #2

answers

Problem 1 Let X be an exponential random variable, with $\lambda = .5$. Find the following probabilities.

a. $P(X \geq 1)$

.606

b. $P(X \geq .4)$

.818

c. $P(X \leq .5)$

.221

d. $P(1 \leq X \leq 2)$

.238

Problem 2 When trucks arrive at the Ambassador Bridge, each must be checked by customs agents. The time of an inspection is exponentially distributed, with a mean of 6 minutes. What is the probability that a truck requires more than 10 minutes to be checked?

The probability of this happening is $e^{-\frac{1}{6}10} = .1889$.

Problem 3 Every day a bakery prepares its famous marble rye. A statistically savvy customer determines that daily demand is normally distributed with a mean of 850 loaves and a standard deviation of 90. How many loaves should the bakery make each day if:

a. It wants the probability of running short on any day to be no more than 30%

We need a Z-value of .52, or $.52 = \frac{X-850}{90}$. Therefore, the bakery should produce 897 loaves.

b. it wants the probability of running short on any day to be no more than 10%

Here, we need a Z-value of 1.28, so the bakery should produce 966 loaves.

Problem 4 Apple observes that the battery in the iPhone 5 lasts for an average of 26 hours under normal usage. Apple believes that battery life per charge is normally distributed, with a standard deviation of 5 hours.

a. What is the probability that a battery lasts between 22 and 30 hours on one charge?

.576

b. What is the probability a battery lasts for less than 22 hours on one charge?

.212

c. What is the probability a battery lasts for more than 30 hours on one charge?

.212

Problem 5 Suppose that the weight of airline passengers with luggage is normally distributed with a mean of 210 pounds and a standard deviation of 50 pounds.

a. What is the probability that one randomly selected passenger (with luggage) weighs more than 280 pounds?

.081

b. What is the probability that a randomly selected sample of 10 passengers weigh more than 250 pounds, on average?

omitted

c. Suppose safety regulations require that a plane carry less than 90,000 pounds. Suppose that the airline allows 400 people on the plane. What is the probability that the regulations are met?

omitted

d. Bob Largebody weighs more than 99% of passengers. What is Bob's weight?

Bob's Z-score is about 2.33, meaning he is about 2.33 standard deviations above the mean, or that he weighs 326.5 pounds.

Problem 6 In class, we discussed what fraction of 7-foot or taller males of an appropriate age play in the NBA. This exercise demonstrates the sensitivity of such calculations to underlying assumptions.

a. There are 56 US-born 7-footers who have played at least one game in the NBA since the 1999-2000 season. There are X total 7-footers between the ages of 20 and 35 living in the US. Estimate X under the following assumptions:

- There are 316,710,000 total people living in the US
- 49.22% of people living in the US are male.
- $\frac{1}{3}$ of all males are between the ages of 20 and 35.
- The average height of a male is 5 feet, 10 inches.
- The standard deviation of male height is 3 inches

Under these assumptions, there are about 80 7-foot men of NBA age living in the U.S.

b. Repeat part a, but now assume that the standard deviation of male height is 3.2 inches. How does this change your estimate of the fraction of all 7-footers who play in the NBA?

315 7-foot men in of NBA age in the U.S.

b. Repeat part a, but now assume that the standard deviation of male height is 3.4 inches. How does this change your estimate of the fraction of all 7-footers who play in the NBA?

994 7-foot men in of NBA age in the U.S.

Problem 7 During Eco 391's exams, you will be given a photocopy of certain probability tables from the back of your textbook. To practice using these tables, calculate the following probabilities first in Excel, and then using table 3 in Appendix B. Note that the probability tables give cumulative probabilities, that is $P(X \leq x)$. Use two decimal places in all calculations.

a. Suppose X is a normal random variable with mean -2 and standard deviation 10. Find $P(X < -5)$, $P(X \geq 0)$, and $P(X \geq 10)$.

$P(X < -5) = .758$, $P(X \geq 0) = \frac{1}{2}$, and $P(X \geq 10) = .115$

b. Suppose that X is a normal random variable with mean 100 and standard deviation 6. Find $P(X \leq 110)$, $P(X \geq 85)$, and $P(X \leq 100)$.

$P(X \leq 110) = .9522$, $P(X \geq 85) = .9938$, and $P(X \leq 100) = \frac{1}{2}$

c. Suppose that X is a normal random variable with mean 30 and standard deviation 2. Find $P(X \geq 28)$, $P(27 \leq X \leq 34)$, and $P(X \leq 35)$

$P(X \geq 28) = .841$, $P(27 \leq X \leq 34) = .91$, and $P(X \leq 35) = .994$