

**Midterm #1**

10/2/2013

**Instructions:** Refer to the provided probability tables as necessary. You may use a calculator, and one sheet of notes. You will never be penalized for showing work, but if what is asked for can be computed directly, points awarded will depend primarily on the correctness of your numerical answer. The exam is out of 80 points, so you get 20 points for free. Good luck!

**Problem 1 (10 points)** There is a horrible disease spreading. If you get the disease, it will turn you into a shrubbery. There are two different tests for the disease – one tests your blood and the other tests your saliva. The tests are very rarely in error. You see on the news that the probability that a randomly selected person will test positive for the first test is 0.2 and the probability that a randomly selected person will test positive for the second test is also 0.2.

Your friend intends to have both tests done. Knowing that you are an expert in probability, he asks you: “What is the probability that both tests will turn out positive?”

a. What would the correct answer be if the test results were independent?

b. Do you think that the two test results are actually independent? Explain.

c. Do you think that the right answer to your friend’s question is higher or lower than what you guessed in part a.? Explain in the context of part b.

**Problem 2 (15 points)** Find the following probabilities.

a.  $X$  is a normal random variable, with mean 10 and standard deviation .5. Find  $P(10 \leq X \leq 11)$ .

b.  $X$  is a uniform random variable on the interval  $[50, 100]$ . Find  $P(40 \leq X \leq 70)$ .

c.  $X$  is a Poisson random variable, with expected value  $\mu = 5$ . Find  $P(X \geq 3)$ .

**Problem 3 (10 points)** To pass a test, you must correctly answer at least 10 of the 20 questions. You have studied enough so that you can correctly answer any one question with probability  $p$ . Assume that questions are drawn independently from a common pool, so that the probability of a correct answer is the same for every question, but answering one question correctly makes it no more or less likely that you answer another correctly.

a. Find the probability that you pass the exam if  $p = .5$ .

b. Find the probability that you pass the exam if  $p = .6$ .

c. Find the probability that you pass the exam if  $p = .7$ .

d. Based on your answers to a-c, does it appear that the probability of passing is increasing at a constant, increasing or decreasing rate in  $p$ ? What is the intuition?

**Problem 4 (15 points)** You run a mutual fund that holds shares in many firms, but sometimes the firms go bankrupt. Assume that the number of bankruptcies follows a Poisson process with a mean of 2.5 bankruptcies each year.

a. What is the probability of no bankruptcies this year?

b. What is the probability of no bankruptcies for three consecutive years?

c. Suppose the time between successive bankruptcies follows an exponential distribution, with an average time of .4 years. Let  $X$  measure the time between bankruptcies (in years).

i. Suppose a company has just gone bankrupt. What is the probability that it will be more than one year before the next bankruptcy?

ii. What is the probability that it will be more than four years before the next bankruptcy?

**Problem 5 (10 points)** The weight of luggage carried onto a plane by passengers is normally distributed with a mean of 20 KG and a standard deviation of 6 KG.

a. What is the probability that a passenger's luggage weighs 25 KG?

b. Passengers are "fast-tracked" if they have luggage weighing less than 10 KG. What percentage of passengers are fast tracked?

c. The airline wants to set the maximum weight limit so that only 2.5% of passengers have to pay an overweight luggage fee. What limit should it set?

d. If the plane has 100 passengers, what is the probability that the average weight of their luggage is less than 23 KG?

**Problem 6 (10 points)** A new test for Bennin's Disease, which afflicts 1 in 10,000 people, has been developed. If administered to a patient with Bennin's Disease, the probability of a positive test is 99.9%, while if administered to someone who doesn't have the disease, the probability of a (false) positive is 1%. Given a positive test result, what is the probability that patient has Bennin's disease?

**Problem 7 (10 points)** The tables below list the possible outcomes for a random variable  $X$ , but do not give their probabilities. Fill in the probabilities as directed. You may assume that  $X$  does not take values outside of the set  $\{1, 2, 3, 4, 5, 6\}$ .

**a.** Fill in the probabilities in the second column so that the expected value of  $X$  is equal to 4.5. There are many possible correct answers. Show your work.

value	probability
1	
2	
3	
4	
5	
6	

**b.** Fill in the probabilities in the second column so that the expected value of  $X$  is equal to 2 and the standard deviation of  $X$  is equal to 0. Show your work.

value	probability
1	
2	
3	
4	
5	
6	

**c.** Fill in the probabilities in the second column so that the expected value of  $X$  is equal to 4 and the standard deviation of  $X$  is equal to 1. Show your work.

value	probability
1	
2	
3	
4	
5	
6	

**Bonus Problem (+5 points)** Every day you pass by two police officers checking for speeders. The first police officer catches speeders 1% of the time. The second police officer catches speeders 2% of the time. The two officers act independently of each other. You travel on the highway for 25 days and you speed every day. What is the probability of getting at least one ticket?