

Homework 3

due Wednesday, February 20, in class

Instructions: Complete all problems and turn in a set of answers either to me or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write “According to Excel, ...” in your answer.

Problem 1 Suppose that the weight of airline passengers with luggage is normally distributed with a mean of 210 pounds and a standard deviation of 50 pounds.

- a. What is the probability that a randomly selected sample of 10 passengers weigh more than 250 pounds, on average?
- b. Suppose safety regulations require that a plane carry less than 90,000 pounds. Suppose that the airline allows 400 people on the plane. What is the probability that the regulations are met?

Problem 2 You collect a “sample” of 5 dice rolls. Let \bar{X} equal the mean outcome of the 5 rolls.

- a. Find $P(\bar{X} = 1)$
- b. Find $P(\bar{X} = 6)$
- c. Prove that $P(\bar{X} = 3) > P(\bar{X} = 1)$ (note: one way of doing this problem is to calculate $P(\bar{X} = 3)$, but there is a much simpler way to do it)

Problem 3 A normally distributed population has a mean of 40 and a standard deviation of 12. What does the Central Limit Theorem say about the sampling distribution of the mean if samples of size 100 are drawn from this population?

Problem 4 A sample of $n = 64$ observations is drawn from a population with $\mu = 100$ and $\sigma = 16$. Find the following:

- a. $P(\bar{X} \geq 104)$
- b. $P(\bar{X} \leq 94)$
- c. $P(\bar{X} \geq 110)$

Problem 5 A statistics practitioner took a random sample of 50 observations from a population with a standard deviation of 25 and computed the sample mean to be 100.

- a. Use an interval estimate to estimate the population mean with 90% confidence.
- b. Repeat part a. using a 95% confidence level.
- c. Repeat part a. using a 99% confidence level.

d. Describe the effect on the confidence interval of increasing the confidence level.

Problem 6 The mean of a sample of 25 was calculated as $\bar{x} = 500$. The sample was randomly drawn from a population with a standard deviation of 15.

a. Estimate the population mean with 99% confidence.

b. Repeat part a. changing the population standard deviation to 30.

c. Repeat part a. changing the population standard deviation to 60.

d. Describe the effect on the confidence interval of increasing the population standard deviation.

Problem 7 A medical statistician wants to estimate the average weight loss of people who are on a new diet plan. In a preliminary study, he guesses that the standard deviation of the population of weight losses is about 10 pounds. How large a sample should he take to estimate the mean weight loss to within 2 pounds, with 90% confidence?