Midterm 1

answers

Instructions: Refer to the provided probability tables as necessary. You may use a calculator, and one sheet of notes. You will never be penalized for showing work, but if what is asked for can be computed directly, points awarded will depend primarily on the correctness of your numerical answer. The exam is out of 85 points, so you get 15 points for free. Good luck!

Problem 1 (15 points) One of the first practical applications of the Poisson distribution was modelling the number of Prussian soldiers killed by horse kick. Suppose that 3 soldiers per year are killed by horse kick, and assume X, the number of horse kick deaths in one year follws a Poisson distribution. Find the following probabilities.

a. Find $P(X = 0)$.

.0498

b. Find $P(X \geq 2)$.

.8009

c. Find $P(1 \leq X \leq 6)$.

.9167

Problem 2 (15 points) Let X be an exponential random variable with $\lambda = .2$. Find the following:

a. Find $P(X \geq 5)$.

.3679

b. Find $E(X)$, the expected value of X.

5

c. Find $P(X = 7)$

0

Problem 3 (10 points) The Kentucky Geological Survey is interested in modeling the amount of time before the next earthquake felt in Lexington. They are considering the following distributions: normal, uniform, exponential. Which of these distributions is most appropriate, and why?

Waiting times are typically modeled with exponential random variables. The reason is that the probability of waiting time t falls gradually as t increases, reflecting the fact that lower waiting times are more likely. In other words, it is more likely that the next earthquake will be in the next year than in the following year, and more likely in the following year than the year after that. With a uniform distribution, all time periods of equal length are equally likely, which doesn't make sense. The density of the normal distribution first rises, then falls, which does not seem to fit the earthquake example.

Problem 4 (10 points) X is a normal random variable with mean 10 and standard deviation 1.7.

a. Find $P(X > 14)$.

.00931

b. Find $P(10 < X < 12)$.

.3803

Problem 5 (15 points) A zoologist finds that distance of migratory travel among malagasy warblers (a type of bird) follows a normal distribution, with a mean of 1,800 miles, and a standard deviation of 350 miles. Let X denote the distance traveled by a randomly chose warbler.

a. Find $P(X < 2,000)$.

.7161

b. Find a distance such that 95% of all warblers travel less than this distance.

2375.6988 miles

c. Find a distance such that only 1% of all warblers travel less than this distance.

985.7782 miles

Problem 6 (10 points) A new test for Bennin's Disease, which afflicts 1 in 10,000 people, has been developed. If administered to a patient with Bennin's Disease, the probability of a positive test is 99.9%, while if administered to someone who doesn't have the disease, the probability of a (false) positive is 1%. Given a positive test result, what is the probability that patient has Bennin's disease?

Applying Bayes' rule,

$$P(\text{has disease}|\text{positive test}) =$$
$$\frac{P(\text{has disease}) * P(\text{tests positive}|\text{has disease})}{P(\text{has disease}) * P(\text{tests positive}|\text{has disease}) + P(\text{does not have disease}) * P(\text{tests positive}|\text{does not have disease})}$$
$$= \frac{\frac{1}{10000} * .999}{\frac{1}{10000} * .999 + \frac{9999}{10000} * .01}$$
$$= .009892$$

So there is less than a 1% chance that a patient who tests positive has Bennin's disease.

Problem 7 (10 points) In an attempt to cheat his friends at games of chance, Bob constructs special weighted dice so that the number that comes up when a die is rolled follows the probability distribution below:

value	probability
1	.6
2	0
3	.2
4	.1
5	.05
6	.05

a. If X is the outcome of a roll of one die, calculate the expected value and standard deviation of X.

The expected value is 2.15, the standard deviation is 1.558

b. If two dice are rolled simultaneously, calculate the probability the total is a 4.

.24