Homework #2

answers

Problem 1 Let X be an exponential random variable, with $\lambda = .5$. Find the following probabilities.

a. $P(X \geq 1)$

.606

b. $P(X \geq .4)$

.818

c. $P(X \leq .5)$

.221

d. $P(1 \leq X \leq 2)$

.238

Problem 2 Find the following probabilities.

a. X is a normal random variable, with mean 10 and standard deviation .5. Find $P(10 \leq X \leq 11)$.

.477

b. X is a uniform random variable on the interval $[50, 100]$. Find $P(40 \leq X \leq 70)$.

$\frac{2}{5}$

c. X is a Poisson random variable, with expected value $\mu = 5$. Find $P(X \geq 3)$.

.875

Problem 3 The weight of luggage carried onto a plane by passengers is normally distributed with a mean of 20 KG and a standard deviation of 6 KG.

a. What is the probability that a passenger's luggage weighs 25 KG?

.2023

b. Passengers are "fast-tracked" if they have luggage weighing less than 10 KG. What percentage of passengers are fast tracked?

.0478

c. The airline wants to set the maximum weight limit so that only 2.5% of passengers have to pay an overweight luggage fee. What limit should it set?

31.7598 KG

d. If the plane has 100 passengers, what is the probability that the average weight of their luggage is less than 23 KG?

The probability is very close to 1 (the exact answer is 0.99999971335, but any answer that notes that the probability is very small close to one will receive full points).

Problem 4 Every day a bakery prepares its famous marble rye. A statistically savvy customer determines that daily demand is normally distributed with a mean of 850 loaves and a standard deviation of 90. How many loaves should the bakery make each day if:

a. It wants the probability of running short on any day to be no more than 30%
We need a Z-value of .52, or $.52 = \frac{X-850}{90}$. Therefore, the bakery should produce 897 loaves.

b. it wants the probability of running short on any day to be no more than 10%
Here, we need a Z-value of 1.28, so the bakery should produce 966 loaves.

Problem 5 The amount of time spent by American adults watching TV per day is normally distributed with a mean of 6 hours and a standard deviation of 1.5 hours.

a. What is the probability that a randomly selected American adult watches television for more than 7 hours per day?

.252

b. What is the probability that the average time spent watching TV in a random sample of 5 American adults is more than 7 hours?

.068

c. What is the probability that the average time spent watching TV in a random sample of 30 American adults is more than 7 hours?

.00013

Problem 6 In class, we discussed what fraction of 7-foot or taller males of an appropriate age play in the NBA. This exercise demonstrates the sensitivity of such calculations to underlying assumptions.

a. There are 20 US-born 7-footers who have played at least one game in the NBA in the current 2013-2014 season. There are X total 7-footers between the ages of 20 and 35 living in the US. Estimate X under the following assumptions:

- There are 316,710,000 total people living in the US
- 49.22% of people living in the US are male.
- $\frac{1}{4}$ of all males are between the ages of 20 and 35.
- The average height of a male is 5 feet, 10 inches.
- The standard deviation of male height is 3 inches

Given these assumptions, and the assumption that height is normally distributed, the probability that any adult male is over 7 feet tall is .00000153. Multiplying this probability by the number of 20-35 year old adult males gives an estimate of $X = 59.65$. This would mean that about $\frac{1}{3}$ of 20-35 year old 7-footers currently play int he NBA.

b. Repeat part a, but now assume that the standard deviation of male height is 3.2 inches. How does this change your estimate of the fraction of all 7-footers who currently play in the NBA ($\frac{20}{X}$)?

This changes our estimate of X to 236.6, meaning that about 8.5% of 20-35 year old males currently play in the NBA.

b. Repeat part a, but now assume that the standard deviation of male height is 3.4 inches. How does this change your estimate of the fraction of all 7-footers who play in the NBA?

This changes our estimate of X to 745.83, meaning that about 2.7% of 20-35 year old males currently play in the NBA.

Problem 7 During Eco 391's exams, you will be given a photocopy of certain probability tables from the back of your textbook. To practice using these tables, calculate the following probabilities first in Excel, and then using table 3 in Appendix B. Note that the probability tables give cumulative probabilities, that is $P(X \leq x)$. Use two decimal places in all calculations.

a. Suppose X is a normal random variable with mean -2 and standard deviation 10. Find $P(X < -5)$, $P(X \geq 0)$, and $P(X \geq 10)$.

$P(X < -5) = .382$, $P(X \geq 0) = 0.4207$, and $P(X \geq 10) = 0.115$

b. Suppose that X is a normal random variable with mean 100 and standard deviation 6. Find $P(X \leq 110)$, $P(X \geq 85)$, and $P(X \leq 100)$.

$P(X \leq 110) = .9522$, $P(X \geq 85) = .9938$, and $P(X \leq 100) = \frac{1}{2}$

c. Suppose that X is a normal random variable with mean 30 and standard deviation 2. Find $P(X \geq 28)$, $P(27 \leq X \leq 34)$, and $P(X \leq 35)$

$P(X \geq 28) = .841$, $P(27 \leq X \leq 34) = .91$, and $P(X \leq 35) = .994$