

## Homework #3

### answers

**Instructions:** Complete all problems and turn in a set of answers either to me, in my mailbox, or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write “According to Excel, ...” in your answer.

**Problem 1** Let  $X$  be a normal random variable, with mean 6 and standard deviation .4.

a. Find  $P(5.40 \leq X \leq 6.92)$

.9225

b. Find  $P(6.56 < X)$

.0808

c. Let  $\bar{X}$  denote the average of 64 draws from  $X$ . Find  $P(5.9 < \bar{X} < 6.1)$ .

.9545

d. Let  $\bar{X}$  denote the average of 100 draws from  $X$ . Find  $P(5.9 < \bar{X} < 6.1)$ .

.9876

**Problem 2** A statistics practitioner took a random sample of 50 observations from a population with a standard deviation of 25 and computed the sample mean to be 100.

a. Use an interval estimate to estimate the population mean with 90% confidence.

[94.185, 105.815]

b. Repeat part a. using a 95% confidence level.

[93.070, 106.930]

c. Repeat part a. using a 99% confidence level.

[90.893, 109.107]

d. Describe the effect on the confidence interval of increasing the confidence level.

A higher confidence level generates a longer interval.

**Problem 3** The mean of a sample of size  $n = 35$  was calculated as  $\bar{x} = 503.4$ . The sample was randomly drawn from a population with a standard deviation of 15. A researcher wishes to perform the following hypothesis test:

$$H_0 : \mu \leq 500$$

$$H_A : \mu > 500$$

a. Determine the t-statistic of the above test.

1.341

b. Determine the p-value of the above test.

.08996

c. Suppose that a larger sample of size  $n = 75$  is taken, and that the sample mean remains  $\bar{x} = 503.4$ . Determine the p-value of the hypothesis test described above.

.024824

d. Repeat part c. for a sample of size  $n = 125$ .

.005635

e. What is the relationship between sample size and the p-value of a test, all else equal?

As sample size increases, p-value decreases, all else equal.

**Problem 4** A medical statistician wants to estimate the average weight loss of people who are on a new diet plan. In a preliminary study, he guesses that the standard deviation of the population of weight losses is about 10 pounds. How large a sample should he take to estimate the mean weight loss to within 2 pounds, with 90% confidence?

The margin of error in his estimate is  $1.645 * \frac{10}{n}$ , where  $n$  is the sample size. Setting this equal to 2 and solving for  $n$  gives us  $n = 67.6$ , so a sample of size 68 is required for the margin of error to be below 2 pounds.

**Problem 5** Fightmaster and Associates Real Estate, Inc. advertises that the mean selling time of a residential home is 40 days or less. A sample of 50 recently sold residential homes shows a sample mean selling time of 34 days and a standard deviation of 20 days. Using  $\alpha = .02$ , test the validity of the company's claim.

Reject the null hypothesis that  $\mu \geq 40$  if and only if the test statistic is less than  $-2.054$ . Given these data, the test statistic comes out to  $-2.12$ , so we reject  $H_0$ .

**Problem 6** Demographers in certain countries are becoming concerned about a "demographic winter" occurring if the average birthrate among women falls below the replacement level of 2. A Belgian demographer takes a sample of 250 women and finds an average birthrate of 1.964, with a standard deviation of 0.3.

a. Test at significance  $\alpha = .05$  whether there is sufficient evidence to conclude that the birthrate in Belgium is below the replacement level.

The test statistic is  $\frac{1.964-2}{\frac{.3}{\sqrt{250}}} = -1.897$ . Since the rejection region for this one-sided hypothesis test is to reject for t-statistics less than 1.64, we reject  $H_0$  in favor of  $H_a$ .

b. What is the p-value of the test in a.?

.02889

**Problem 7** You wish to estimate the average number of times a UK freshman calls home during the fall semester. You conduct a survey of 156 freshman and determine that the sample mean is 29.2, with a standard deviation of 9.14

a. Give a 95% confidence interval for the average number of calls during the fall semester.

[27.7657, 30.6343]

b. Give a 90% confidence interval for the average number of calls during the fall semester.

[27.9963, 30.4037]

c. If the sample standard deviation stays constant, calculate the minimum sample size necessary so that the distance between the lower and upper limits of your 95% confidence interval is 2 or less.

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