Midterm #1

answers

Problem 1 Calculate the following:

a. X follows a Poisson distribution, with a mean of 2.5. Find $P(X \geq 2)$.

.7127

b. X follows a uniform distribution, with lower limit 0 and upper limit 20. Find $P(15 \leq X \leq 20)$.

$\frac{1}{4}$

c. X follows an exponential distribution, with $\lambda = 1$. Calculate $P(X > 1)$.

.3679

Problem 2 The daily milk production of a dairy cow follows a normal distribution, with a mean of 6.5 gallons and a standard deviation of 1.5 gallons. Farmer Dave has just purchased 20 dairy cows, which were chosen at random.

a. What is the probability that more than 15 of the cows will produce at least 7 gallons of milk per day?

The probability that any one cow produces more than 7 gallons of milk is .3694. The number of cows that produce at least this much milk is a binomial random variable (with $n = 20$, $p = .3694$). Although this value of p does not appear in the binomial table, we do have that for $p = .3$, $P(X \leq 14) = 1.0000$, and for $p = .4$, $P(X \leq 14) = .9997$. Since the probability for $p = .3694$ is somewhere in between these two, we have that the probability at least 15 cows produce this much milk is less than .0003, i.e. very close to 0.

b. What is the probability that all of the 20 cows will produce at least 4 gallons of milk/day?

$.9522^{20} = .3755$.

c. What is the probability that the average milk production of the 20 cows will be at least 6 gallons/day?

By the Central Limit Theorem, the distribution of average milk production is normal, with an average of 6.5, and a standard deviation of $\frac{1.5}{\sqrt{20}} = .3354$. Therefore, the probability that average milk production is above 6 is .9320.

Problem 3 The weight of female mountain lions follows a normal distribution, with a mean of 93 pounds, and standard deviation of 20 pounds.

a. What is the probability a randomly chosen female mountain lion weighs more than 100 pounds?

.3632

b. Susie the mountain lion weighs more than $\frac{2}{3}$ of her peers. Approximately how much does Susie weigh?

101.6145 pounds

c. Prof. Puma observes 30 female mountain lions at random, and weighs them.[1] What is the probability that the average weight in the sample is less than 90 pounds?

By the Central Limit Theorem, the average weight of a sample of 30 mountain lions is a normal random variable, with mean 93 and standard deviation $\frac{20}{\sqrt{30}} = 3.6515$. The probability that such a random variable produces a draw below 90 is .2057.

Problem 4 A coin is either fair (heads and tails are equally likely) or fixed (heads always comes up). You initially believe the probability of a fair coin is $\frac{1}{2}$. The coin is then flipped three times, and all come up heads. Given this, what is the probability the coin is fair?

The total probability of three heads is $p(\text{fair}) * p(3 \text{ heads}|\text{fair}) + p(\text{fixed}) * p(3 \text{ heads}|fixed) = .5625$. The probability the coin is fair *and* produces 3 consecutive heads is $p(\text{fair}) * p(3 \text{ heads}|\text{fair}) = .5 * .125 = .0625$. Therefore, the probability the head is fair given 3 consecutive heads is $\frac{.0625}{.5625} = .1111$

Problem 5 You have the opportunity to construct a special 6-sided die, which will follow any probability distribution you choose. For each part, fill in the probabilities as directed.

a. Fill in the probabilities in the second column so that the expected value of a roll of the die is equal to 2.5. There are many possible correct answers. Show your work.

value	**probability**
1	
2	
3	
4	
5	
6	

There are many possible correct answers. The most straightforward is perhaps $P(2) = P(3) = .5$.

b. Fill in the probabilities in the second column so that the probability of a roll of at least 3 is greater than or equal to $\frac{1}{2}$. Show your work.

value	**probability**
1	
2	
3	
4	
5	
6	

There are many correct answers. For example, $P(6) = 1$.

c. Fill in the probabilities in the second column so that the expected value of a roll of the die is equal to 3. $P(1)$ has been done for you. Show your work.

[1] Do not attempt to weigh a mountain lion unless you have been specially trained to do so.

value	probability
1	$\frac{1}{4}$
2	
3	
4	
5	
6	

There are many correct answers. One particularly straightforward answer is $P(1) = P(2) = P(4) = P(5) = \frac{1}{4}$.

Problem 6 The number of puppies in a Rottweiler litter[2] follows a Poisson distribution, with mean 7.5.

a. What is the probability a randomly chosen litter has 10 or more puppies?

.2236

b. The Rottweiler Breeding Association of Kentucky randomly surveys its members and finds that 64 litters are expected in Kentucky in 2014. What is the probability that the average litter size is 10 or more?

Using the Central Limit Theorem, the average litter size is approximately a normal random variable, with mean 7.5 and standard deviation $\frac{\sqrt{7.5}}{\sqrt{64}} = .3423$ (the variance of a Poisson distribution is equal to its mean). Hence, the probability that the average litter size will be 10 or more is the probability of observing a z-score of 7.3035 or more, which is very, very close to zero. As this can't be computed directly using the normal probability table, any answer that points out the probability is close to zero is fine.

c. Referring to part b, what is the probability that the average litter size is 7 or more?

Again, the average litter size is approximately a normal random variable, with mean 7.5 and standard deviation .3423. Hence, the probability that average litter size exceeds 7 is .9280.

Problem 7 On any given day, there is a 20% chance that Professor Gonzo forget to show up to teach his class. If there are 25 scheduled meetings throughout the semester, what is the probability he teaches at least 20 classes (assume his attendance is independent across classes)?

.6167

Problem 8 The number of daily newspapers sold by a newstand is normally distributed with mean 1,000 and standard deviation 100. How many papers should the stand stock so that it sells out on no more than 40% of days?

1026 newspapers.

Bonus Problem (+5 points) Every day you pass by two police officers checking for speeders. The first police officer catches speeders 1% of the time. The second police officer catches speeders 2% of the time. The two officers act independently of each other. You travel on the highway for 25 days and you speed every day. What is the probability of getting at least one ticket?

[2] "litter" means "group of puppies".

First, we must determine the probability of being stopped on any one given day. There is a 99% chance of getting by the first policeman unmolested, and a 98% chance of getting by the second. Therefore, the probability of getting a ticket on any given day is $1 - .99 * .98 = .0298$. Now, what is the probability of getting at least one ticket in 25 days? First, the probability of getting zero tickets for 25 consecutive days is $P(0\text{ tickets}) = (1 - .0298)^{25} = .4694$, and so the probability of getting at least one ticket is one minus this, or .5306.