# Homework #3

due Tuesday, 10/11/11, in class

**Problem 1** Consider the following simultaneous-move game:

| | | Player 2: Y | Player 2: Z |
|---|---|---|---|
| Player 1 | W | a,b | c,d |
| | X | e,f | g,h |

**a.** List all inequalities that must hold for $(W, Y)$ to be a dominant strategy equilibrium. (hint: an example of an inequality would be $g \geq h$)

$a > e$, $c > g$, $b > d$, $f > h$.

**b.** List all inequalities that must hold for $(W, Y)$ to be a Nash equilibrium.

$a > e$, $b > d$.

**Problem 2** Consider the following simultaneous-move game:

| | | 2: X | Y | Z |
|---|---|---|---|---|
| | A | 4,4 | 10,1 | 0,3 |
| 1 | B | 1,10 | 4,4 | 9,7 |
| | C | 3,0 | 7,9 | 4,4 |

**a.** Find the pure strategy Nash equilibria, if any. There is one pure strategy Nash equilibrium, $(A, X)$.

**b.** There is a mixed strategy equilibrium where player 1 randomizes between A and C, and where player 2 randomizes between X and Z. Solve for this equilibrium.

For player 1 to be indifferent between A and C, given that 2 is mixing between X and Z, we need $4x + 0(1-x) = 3x + 4(1-x)$, where $x$ is the probability with which player 2 plays X. Similarly, for player 2 to be indifferent between X and Z, we need $4a = 3a + 4(1-a)$, where $a$ is the probability with which 1 plays $A$. Solving these two equations gives us $x = a = \frac{4}{5}$, so the mixed strategy equilibrium is located where 1 plays $\frac{4}{5}A + \frac{1}{5}C$ and 2 plays $\frac{4}{5}X + \frac{1}{5}Z$.

**c.** (optional, 5 points extra credit) There is a mixed-strategy equilibrium where both players randomize between all three strategies. Solve for this equilibrium.

**Problem 3** This problem demonstrates a seeming peculiarity about mixed strategy Nash equilibria. Consider the following game between the Chicago Bears' offense and the Detroit Lions' defense. Payoffs are the number of yards advanced (positive yards for Chicago are negative yards for Detroit).

| | | Detroit: run defense | Detroit: pass defense |
|---|---|---|---|
| Chicago | run | -2,2 | 5,-5 |
| | pass | 15,-15 | 1,1 |

**a.** Find all pure strategy Nash equilibria, if any. Then find the mixed-strategy Nash equilibrium of the game.

There are no pure strategy Nash equilibria. There is a mixed-strategy Nash equilibrium where Chicago runs fraction $\frac{16}{23}$ of the time, and passes $\frac{7}{23}$ of the time, and Detroit plays a run defense fraction $\frac{4}{17}$ of the time, and a pass defense $\frac{13}{17}$ of the time.

**b.** Now suppose that the Bears improve their run game by bringing Mike Ditka[1] out of retirement:

| | | Detroit | |
|---|---|---|---|
| | | run defense | pass defense |
| Chicago | run | -2,2 | 10,-10 |
| | pass | 15,-15 | 1,1 |

Find the mixed-strategy Nash equilibrium of the new game.

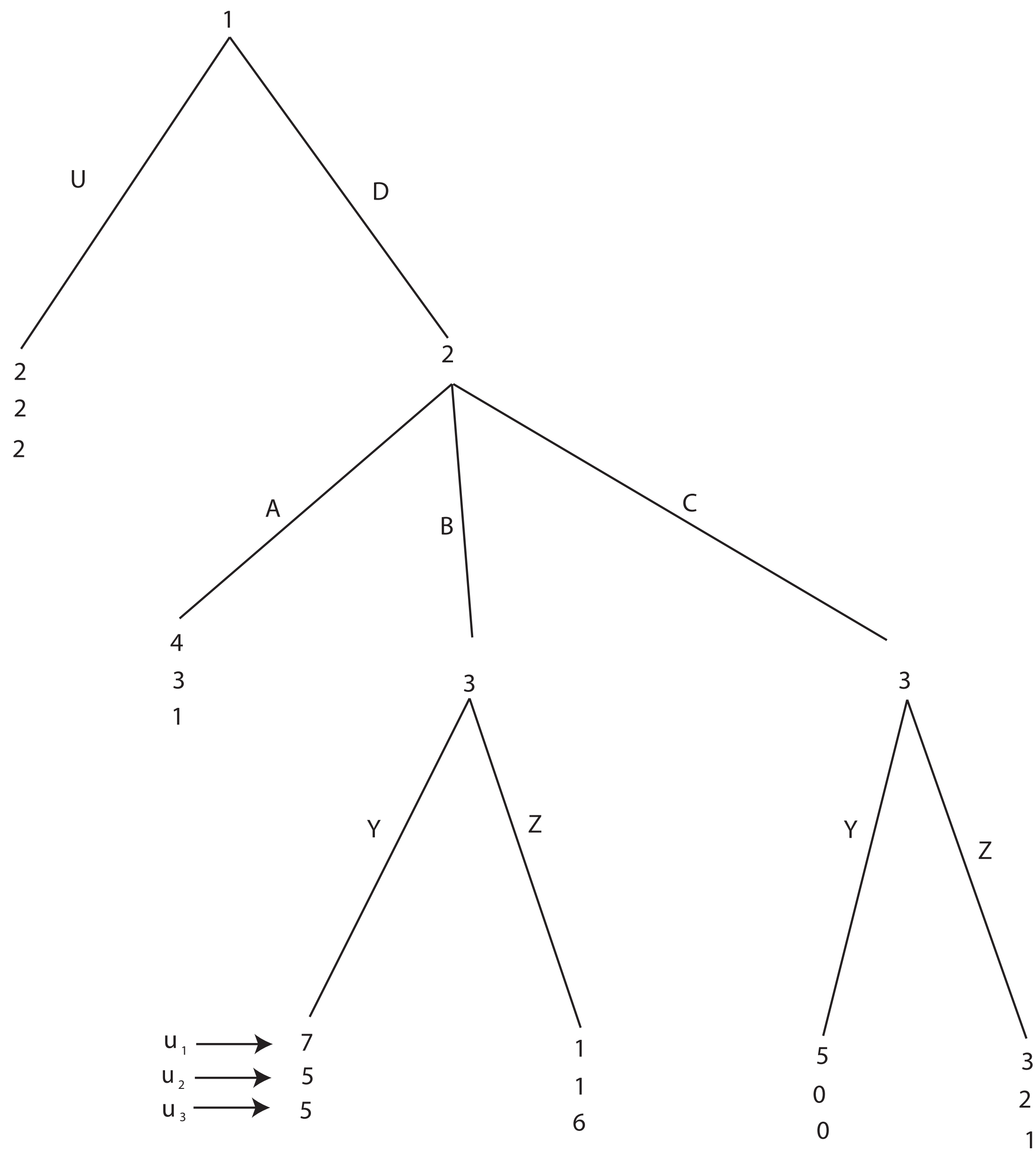
Now Chicago runs fraction $\frac{4}{7}$ of the time, and passes $\frac{3}{7}$ of the time, while Detroit plays a run defense fraction $\frac{9}{26}$ of the time, and a pass defense fraction $\frac{17}{26}$ of the time.

**c.** When running the football becomes a more attractive option for the Bears, do they run more often, or pass more often? Can you explain why?

The Bears run less often when their running game improves. The reason is that after Mike Ditka signs with the Bears, the Lions will become relatively more inclined to play a run defense, lowering the Bear's yardage on run plays.


[1]While Ditka played tight end, the combination of his blocking and the downfield threat he poses as a receiver, even at 71, would help their running game immeasurably.

**Problem 4** Consider the sequential move game below:




What is the equilibrium outcome of this game?

1 plays D, 2 plays A, and 3 plays Z at both nodes (though the game ends before player 3 has a chance to make his decisions). Payoffs are 4, 3, 1.

**Problem 5** Hippos are, from birth, either hawkish or dovish. Hawkish hippos are more likely to have hawkish babies, and dovish hippos are more likely to have dovish babies, though there is, of course, some chance of a parent of one type having a child of the other. Several times a day, an hippo comes across another hippo, and both hippos benefit/are harmed by the interaction as follows:

| | | hippo 2 Hawk | Dove |
|---|---|---|---|
| hippo 1 | Hawk | -12,-12 | 9,0 |
| | Dove | 0,9 | 7,7 |

Think of the payoffs here as gain/loss to reproductive fitness. High payoffs are correlated with high fertility, low payoffs with low fertility.

**a.** Suppose most hippos are hawks. Will hawks or doves reproduce more? Explain why.

Suppose all other hippos are hawks. Then, a dovish hippo will have a payoff of 0, while a hawkish hippo will have a payoff of -2 from each interaction with another hippo. Therefore, doves will reproduce more.

**b.** Suppose most hippos are doves. Will hawks or doves reproduce more? Explain why.

Suppose all other hippos are doves. Then, a dovish hippo will have a payoff of 7, while a hawkish hippo will have a payoff of 9 from each interaction with another hippo. Therefore, hawks will reproduce more.

**c.** Solve for the fraction of hawkish hippos that would result in a stable Nash equilibrium, meaning that hawks and doves reproduce at the same rate.

$\frac{1}{7}$ hawks, $\frac{6}{7}$ doves.

**d.** Explain how an evolutionary process would result in the stable outcome you identify in part c.

If there are more than $\frac{1}{7}$ hawks, doves will reproduce more, as hawks will expend too much energy fighting one another, and so over time, there will be a higher fraction of doves in the hippo population. If there are fewer than $\frac{1}{7}$ hawks, hawks will reproduce more than doves, as a hawkish hippo will be relatively free to simply take all the food and mates he wants, so over time there will be a higher fraction of hawkish hippos.

**e.** Suppose that over time, hippos evolve shorter teeth and claws, so that the payoffs in the (Hawk, Hawk) box change from $(-12, -12)$ to $(-5, -5)$. All other payoffs are the same. Is this adaptation likely to benefit or harm the hippo population? (Hint: is the average fitness of an hippo higher or lower once a new equilibrium between hawks and doves evolves?)

The new evolutionarily stable equilibrium is $\frac{2}{7}$ hawks, and $\frac{5}{7}$ doves. To compare overall fitness, note that fraction $\frac{2}{7} * \frac{2}{7} = \frac{4}{49}$ of all hippo-hippo interactions are between 2 doves, fraction $\frac{5}{7} * \frac{5}{7} = \frac{25}{49}$ are between two doves, and the remainder (fraction $\frac{20}{49}$) are between one dove and one hawk. In a hawk-hawk interaction, each hippo gets payoff $-12$, in a dove-dove interaction, each hippo gets payoff 7, and in a hawk-dove interaction, one hippo gets 0, the other 9. Therefore, the *average* payoff across the entire hippo population is $\frac{4}{49} * (-12) + \frac{25}{49} * 7 + \frac{20}{49} * 4.5 = 4.43$. Average payoff when hippos had longer teeth is 6. Conclude that the adaptation *lowers* overall hippo fitness, despite making hippo fights less deadly. The reason is that hippo fitness is increasing in the fraction of doves, but shorter claws and teeth make hawk-hawk fights less deadly, and so increase the fitness of hawkish hippos.

**Problem 6** Two bills are being considered in Congress (bill A, which would reinstitute the Volstead Act, and bill B, which would prohibit anyone of Canadian origin from owning property). Here are the payoffs to Congress and the president depending upon which laws are passed:

| Outcome | Congress | President |
|---|---|---|
| Bill A only | 8 | -1 |
| Bill B only | -1 | 9 |
| Both bills | 5 | 5 |
| Neither bill | 0 | 0 |

a. Suppose that Congress first decides which of the four options to select. The president can then either sign or *veto*, in which case no law is passed. Which bills become laws in the equilibrium of this sequential game? Explain, with aid of a diagram.

See the diagram on the last page. In the equilibrium, Congress passes both bills, and the President signs them.

b. Now suppose that the president has a *line-item veto*, so that if Congress passes both bills, he can choose to sign bill A or bill B only. However, he cannot enact laws that Congress does not pass. Which bills become laws in the equilibrium of this game? Explain.

Now, if both bills are sent to the President together, he will veto Bill A, but not Bill B. As such, the payoffs to Congress passing both bills are now $(-1, 9)$. The game is otherwise the same as in part a. In tehprobab equilibrium, neither bill becomes law.

c. It is often suggested that giving the president a line-item veto would be a good way to make government work more efficiently, as then he would not have to veto entire bills just because he felt one provision of the bill would make a bad law. In light of this question, what do you think of this suggestion?

A line-item veto would have the direct effect of allowing the President to eliminate parts of legislation he views as harmful, but would have the indirect effect of hindering compromise; under a line-item veto, only bills that both Congress and the President agree on can become law, and it may be difficult to work out a compromise where each side contributes his own favored policies to a bill.