Homework 2

answers

Instructions: Complete all 8 problems. Answers may be handwritten or typed. Students may work together, but must independently write their own answers. Failure to do so will result in a grade of zero. Please turn in your homework to me or under the door of my office (335L) by noon on the assigned due date.

Problem 1 Suppose that most people will not speed if the expected fine is greater than $200. Given current police practices, the probability of being caught speeding is 20%.

a. How high must the actual fine for speeding (what you have to pay if you get a ticket) be to deter most people from speeding?

The expected fine is equal to the probability of being caught times the actual fine, so we have $\$200 = .2 * X$, or $X = \$1000$.

b. Suppose the Lexington mayor caps the amount police can fine speeding motorists at $500. The police vow to step up enforcement in order to cintinue to deter speeding. How high must the probability of being caught speeding be in order to deter most people from speeding?

Here, $\$200 = p * \$500$, or $p = .4$. Police must catch speeders 40% of the time to deter most speeders.

Problem 2 Suppose an investor is considering a business opportunity that would cost $100, but which would generate a return according to the probability distribution below:

Probability	Return
0.2	$100
0.3	$30
0.2	-$10
0.3	-$30

a. What is the expected return of the uncertain investment? What is the variance? What is the standard deviation?

The expected value is $.2 * 100 + .3 * 30 + .2 * -10 + .3 * -30 = \$18$. The variance is $.2 * (100 - 18)^2 + .3 * (30 - 18)^2 + .2 * (-10 - 18)^2 + .3 * (-30 - 18)^2 = 2,236$. The standard deviation is $sqrt2,236 = 47.29$.

b. Would a risk neutral investor take this business opportunity, if he has no alternative investment in mind? What about a risk averse investor?

The investment offers a positive expected return, so a risk neutral investor would take it regardless of the variance. However, a risk averse investor may or may not take the investment, as while doing so pays him a positive expected return, he would prefer to avoid the risk associated with obtaining that return. So, whether or not a risk averse investor would take the opportunity depends on his degree of risk aversion.

Problem 3 Answer the following questions about expected utility and risk aversion:

a. True/false/uncertain: Rex, who is risk averse, prefers a salaried job that will pay him $100,000 with certainty to a commission-based job that will pay him $20,000 with probability .5 and $200,000 with probability .5. (Explain why you think it is true, false, or uncertain.)

Uncertain. The expected salary from the commissioned job is \$110,000, but it is risky. The safe job has a (certain) salary of \$100,000. If Rex is only slightly risk-averse, the increase in expected salary will be enough to compensate him for the extra risk, but if he is very risk averse, he'll definitely prefer the safe job. If it helps, note that someone who is extremely (infinitely) risk averse makes decisions based only on the worst possible outcome, however unlikely that outcome may be, while someone on the other extreme (risk neutral) cares only about the average outcome.

b. Kyle is an English PhD student; there is some chance he will get a prestigious university job that will pay him \$80,000/year (probability p). If he does not get this job, he will have to take a job at Starbuck's, paying \$25,000/year (probability $(1-p)$). Kyle's utility function over salary is given by $\sqrt{w}$, where w is the amount of his salary. Before Kyle has a chance to finish his PhD, he is offered a job at Dunder Mifflin paying \$60,000/year. How low does p have to be before Kyle is better off taking the Dunder Mifflin job? (hint: p is some number between 0 and 1. For example, $p = .25$ means there is a 25% chance of Kyle getting the university job and a 75% chance of his working at Starbuck's.)

Kyle's utility from the Dunder-Mifflin job is $\sqrt{60,000}$, while his utility from waiting to graduate is $p\sqrt{80,000} + (1-p)\sqrt{25,000}$. The former is larger if $p \leq .696$. Therefore, if Kyle thinks there is less than a 69% chance Kyle gets the unviersity job, he'll leave school for the Dunder-Mifflin job.

Problem 3 A driver faces a 5% probability that his car will be in an accident and will be worth nothing. Consider three drivers with cars that have value \$30,000. Abdulla's utility function over the value of his car W is $u(W) = ln(1+W)$. Bedriya's utility function is $u(W) = 100 + 0.5W$.

a. What is Abdulla's risk premium?

Abdulla's expected utility is $EU = .05 * ln(1) + .95 * ln(30,001) = 9.79$. His certainty equivalent is $ln(CE+1) = 9.79$, or $CE = \$17853.3$. As his expected wealth is $0 * .05 + \$30,000 * .95 = \$28,500$, his risk premium is \$10,646.7.

b. What is Bedriya's risk premium?

Bedriya is risk neutral, so her risk premium is 0 (try calculating it following the model in part a).

c. Which of these two people is less likely to take on risk? Which is more likely? How do you know?

Abdulla is less likely to take on risk. For one, he has a higher risk premium. This reflects the fact that he is risk averse, while Bedriya is risk neutral.

Problem 4 The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of of technical substitution of labor for capital is $\frac{1}{4}$. What is the marginal product of capital?

$MRTS = \frac{MPL}{MPK}$, so $MPK = 200$.

Problem 5 The production function for the bicycles of BIKE, Inc. is given by:

$$Q = 10K^{.5}L^{.5}$$

where Q is the number of bicycles produced per day, K is hours of machine time, and L is hours of labor input. BIKE's competitor, PEDAL, Inc., is using the production function:

$$Q = 10K^{.6}L^{.4} \tag{1}$$

a. If both BIKE and PEDAL an equal amount of labor and capital (say, $L = K = X$, where X is any number), which firm will generate more output?

In this case, both firms will have equal output.

b. Assume that, for both BIKE and PEDAL, capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

The MPL at BIKE is $MPL = \frac{15}{\sqrt{L}}$ while the MPL at PEDAL is $\frac{14.95}{L^{.6}}$. It turns out that so long as $L > .97$, the MP is higher at BIKE. The easiest way to see this is to pick values of L and see at which company output increases by more when L increases to $L + 1$. A numerical example is fine to solve this problem.

Problem 6 Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:

# of workers	# of chairs
1	10
2	18
3	24
4	28
5	30
6	28
7	25

a. Calculate the marginal and average product of labor for this production function.

# of workers	MPL	APL
1	10	10
2	8	9
3	6	8
4	4	7
5	2	6
6	-2	4.67
7	-3	3.57

b. Does this production function exhibit diminishing returns to labor? Explain.

Yes, there are diminishing returns to labor; MPL decreases as L increases.

c. Explain intuitively what might cause the marginal product of labor to become negative.

With capital fixed, increasing labor means more and more workers must use the same capital; at some point, there will be too many workers dependent on the same set of machines, etc, to be productive.

Problem 7 The long-run production function for a firm's product is given by $q = f(K, L) = 5 * K * L$. The price of capital is \$10 and the price of labor is \$15.

a. Suppose the firm wishes to produce output of 500. List 5 combinations of capital and labor that the firm can transform into 500 output.

$(K, L) = (100, 1), (20, 5), (10, 10), (4, 25), (2, 50)$. There are many more such combinations.

b. For each of your 5 combinations from part a, give the cost of using that combination of capital and labor. Which is the lowest?

For example, for $(25, 4)$, the cost would be $\$10 * 25 + \$15 * 4 = \$310$. Similar for other input combinations.

c. For your lowest cost combination from part b, calculate the marginal product of capital (MPK) and the marginal product of labor (MPL).

For example, for $(25, 4)$, calculate the MPK by calculating $f(26, 4) = 520$ and calculate MPL by calculating $f(25, 5) = 625$. Therefore, $MPK = 20$ and $MPL = 125$.

d. For your answer in parts b-c, is your marginal product per dollar equal across the two inputs? If not, should the firm use more labor-intensive production or more capital-intensive production?

For $(25, 4)$, $\frac{MPK}{r} = 2$, while $\frac{MPL}{w} = 8.3$, so no, they are not equal. The firm could save money and get the same output by using more labor and less capital.

Problem 8 A firm has production function $f(K, L) = \sqrt{KL}$. In the short run, the firm has capital $K = 400$; this cannot be changed in the near future. The cost of a unit of capital is \$20, while the cost of a unit of labor is \$30.

a. In the short-run, how much labor does the firm need to employ in order to produce $q = 300$ output? What is the cost of producing 300 output?

With 400 capital, the firms needs L labor to produce $q = 20\sqrt{L}$ output, so setting $q = 300$ yields $L = 225$. Therefore, the cost is $225 * \$30 + 400 * \$20 = \$14,750$.

b. Repeat part a for $q = 400$, $q = 500$ and $q = 600$.

The cost of $q = 400$ is \$20,000, the cost of $q = 500$ is \$26,750, the cost of $q = 600$ is \$35,000.

c. What is the cost of producing q output in the short-run?

Producing q output requires $L = \frac{q^2}{400}$ labor. Including the fixed cost of the 400 capital (\$8,000), the total cost is $C(q) = 8,000 + \frac{3}{40}q^2$.

d. What is the marginal cost of producing a 301^{st} unit? A 401^{st} unit? If you are comfortable doing so, you may answer this question by writing down the marginal cost function directly, rather than recalculating total cost for $q = 301$ and $q = 401$.

To calculate MC directly, plug 300 and 301 into the above cost function and take the difference. To calculate it using slopes, MC is the derivative of the cost function, of $MC = \frac{3}{20}q$.

e. Given your answer to part c, draw a graph with the firm's average total cost, average variable cost, and marginal cost (hint: $MC = \frac{3}{20}q$).