

## Homework #4

due Wednesday, 10/24/12, by noon (place under my door)

**Problem 1** Consider the following simultaneous-move game:

		Player 2	
		Y	Z
Player 1	W	a,b	c,d
	X	e,f	g,h

- a. List all inequalities that must hold for  $(W, Y)$  to be a dominant strategy equilibrium. (hint: an example of an inequality would be  $g \geq h$ )
- b. List all inequalities that must hold for  $(W, Y)$  to be a Nash equilibrium.

**Problem 2** This problem demonstrates a seeming peculiarity about mixed strategy Nash equilibria. Consider the following game between the Chicago Bears' offense and the Detroit Lions' defense. Payoffs are the number of yards advanced (positive yards for Chicago are negative yards for Detroit).

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	5,-5
	pass	15,-15	1,1

- a. Find all pure strategy Nash equilibria, if any. Then find the mixed-strategy Nash equilibrium of the game.
- b. Now suppose that the Bears improve their run game by bringing Mike Ditka<sup>1</sup> out of retirement:

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	10,-10
	pass	15,-15	1,1

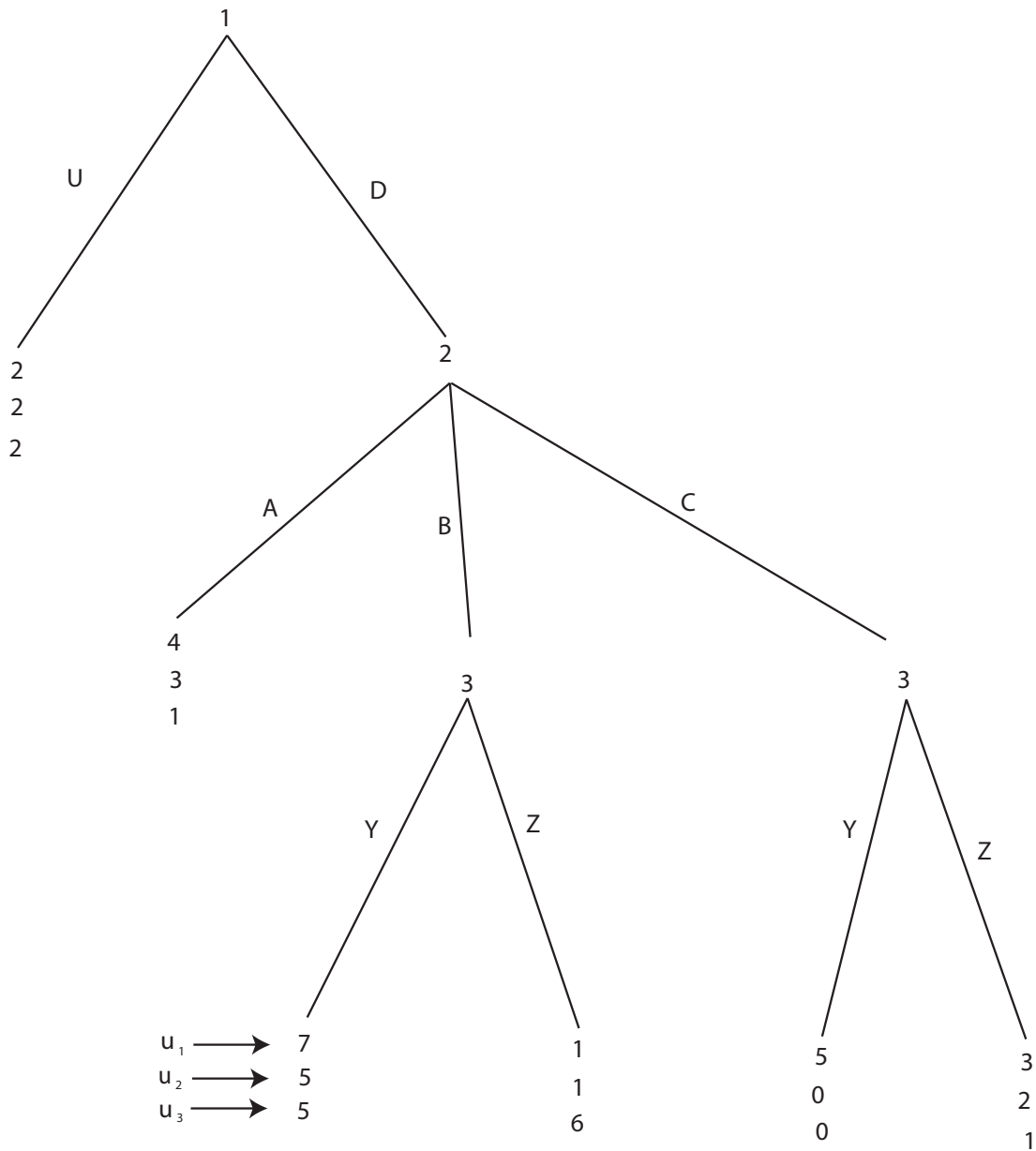
Find the mixed-strategy Nash equilibrium of the new game.

- c. When running the football becomes a more attractive option for the Bears, do they run more often, or pass more often? Can you explain why?

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<sup>1</sup>While Ditka played tight end, the combination of his blocking and the downfield threat he poses as a receiver, even at 71, would help their running game immeasurably.

**Problem 3** Consider the sequential move game below:



What is the equilibrium outcome of this game?

**Problem 4** Consider the following game played between a taxpayer and the IRS:

		IRS	
		Audit	Don't audit
Taxpayer	Cheat on taxes	-60,60	-5,5
	Don't Cheat	-30,25	-30,30

The game has no pure strategy Nash equilibria, but has one mixed strategy Nash equilibrium. Find it.

**Problem 5** Consider the following interaction between two entrepreneurs (players 1 and 2) who are working on a joint project, and a venture capitalist (player 3) who is a potential investor in the project. First, player 1 decides whether to devote high or low effort to preliminary work on the project. Player 2 observes this choice and then decides whether to devote high or low effort himself. They then make a presentation to the venture capitalist, who can observe which, if any, of the entrepreneurs devoted high effort to the project, and decides whether or not to invest.

The payoffs are as follows. Each entrepreneur gets a payment of 5 if the venture capitalist invests and 0 otherwise. In addition, choosing high effort costs an entrepreneur 1, while choosing low effort is free. Investing costs the venture capitalist 2, but if he invests he gains 3 for each entrepreneur who chose high effort. If the venture capitalist does not invest, his payoff is 0. Draw the game tree corresponding to this game and find its equilibrium outcome by solving backwards.

**Problem 6** Consider the game below:

		Man	
		Threaten	Don't threaten
Thief	Steal	30,30	50,35
	Don't steal	40,60	20,20

- a. What is the Nash equilibrium if choices are made simultaneously?
- b. What is the equilibrium outcome if player 1 chooses first?
- c. What is the equilibrium outcome if player 2 chooses first?

**Problem 7** Two bills are being considered in Congress (bill A, which would reinstitute the Volstead Act, and bill B, which would prohibit anyone of Canadian origin from owning property). Here are the payoffs to Congress and the president depending upon which laws are passed:

Outcome	Congress	President
Bill A only	8	-1
Bill B only	-1	9
Both bills	5	5
Neither bill	0	0

- a. Suppose that Congress first decides which of the four options to select. The president can then either sign or *veto*, in which case no law is passed. Which bills become laws in the equilibrium of this sequential game? Explain, with aid of a diagram.
- b. Now suppose that the president has a *line-item veto*, so that if Congress passes both bills, he can choose to sign bill A or bill B only. However, he cannot enact laws that Congress does not pass. Which bills become laws in the equilibrium of this game? Explain.
- c. It is often suggested that giving the president a line-item veto would be a good way to make government work more efficiently, as then he would not have to veto entire bills just because he felt one provision of the bill would make a bad law. In light of this question, what do you think of this suggestion?