

Homework #4

answers

Problem 1 Consider the following simultaneous-move game:

		Player 2	
		Y	Z
Player 1	W	a,b	c,d
	X	e,f	g,h

a. List all inequalities that must hold for (W, Y) to be a dominant strategy equilibrium. (hint: an example of an inequality would be $g \geq h$)

$$a > e, c > g, b > d, f > h.$$

b. List all inequalities that must hold for (W, Y) to be a Nash equilibrium.

$$a > e, b > d.$$

Problem 2 This problem demonstrates a seeming peculiarity about mixed strategy Nash equilibria. Consider the following game between the Chicago Bears' offense and the Detroit Lions' defense. Payoffs are the number of yards advanced (positive yards for Chicago are negative yards for Detroit).

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	5,-5
	pass	15,-15	1,1

a. Find all pure strategy Nash equilibria, if any. Then find the mixed-strategy Nash equilibrium of the game.

There are no pure strategy Nash equilibria. There is a mixed-strategy Nash equilibrium where Chicago runs fraction $\frac{16}{23}$ of the time, and passes $\frac{7}{23}$ of the time, and Detroit plays a run defense fraction $\frac{4}{17}$ of the time, and a pass defense $\frac{13}{17}$ of the time.

b. Now suppose that the Bears improve their run game by bringing Mike Ditka¹ out of retirement:

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	10,-10
	pass	15,-15	1,1

Find the mixed-strategy Nash equilibrium of the new game.

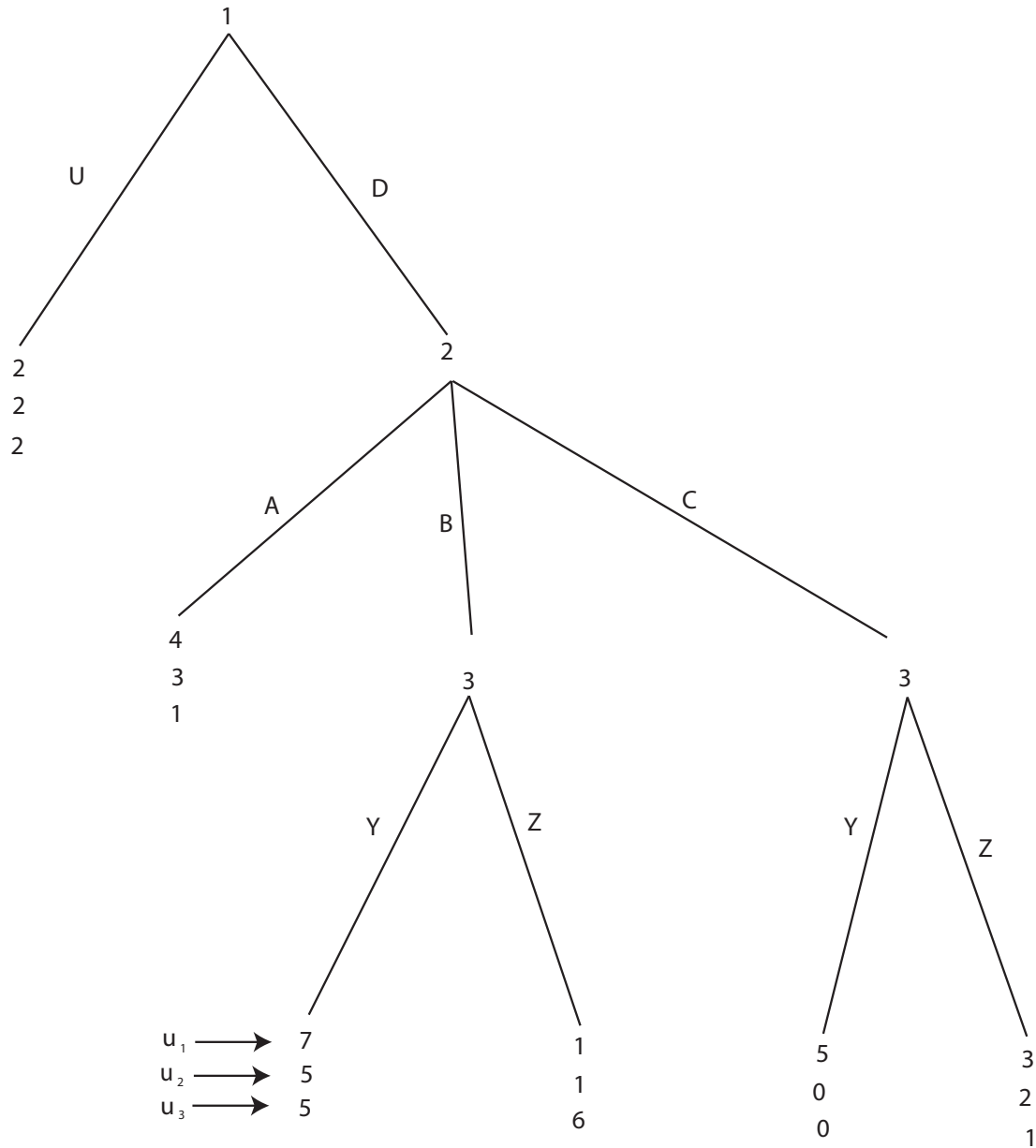
Now Chicago runs fraction $\frac{4}{7}$ of the time, and passes $\frac{3}{7}$ of the time, while Detroit plays a run defense fraction $\frac{9}{26}$ of the time, and a pass defense fraction $\frac{17}{26}$ of the time.

c. When running the football becomes a more attractive option for the Bears, do they run more often, or pass more often? Can you explain why?

¹While Ditka played tight end, the combination of his blocking and the downfield threat he poses as a receiver, even at 71, would help their running game immeasurably.

The Bears run less often when their running game improves. The reason is that after Mike Ditka signs with the Bears, the Lions will become relatively more inclined to play a run defense, lowering the Bear's yardage on run plays.

Problem 3 Consider the sequential move game below:



What is the equilibrium outcome of this game?

1 plays D, 2 plays A, and 3 plays Z at both nodes (though the game ends before player 3 has a chance to make his decisions). Payoffs are 4, 3, 1.

Problem 4 Consider the following game played between a taxpayer and the IRS:

		IRS	
		Audit	Don't audit
Taxpayer	Cheat on taxes	-60,60	-5,5
	Don't Cheat	-30,25	-30,30

The game has no pure strategy Nash equilibria, but has one mixed strategy Nash equilibrium. Find it.

The taxpayer cheats $\frac{1}{12}$ of the time, the IRS audits 11 of the time.

Problem 5 Consider the following interaction between two entrepreneurs (players 1 and 2) who are working on a joint project, and a venture capitalist (player 3) who is a potential investor in the project. First, player 1 decides whether to devote high or low effort to preliminary work on the project. Player 2 observes this choice and then decides whether to devote high or low effort himself. They then make a presentation to the venture capitalist, who can observe which, if any, of the entrepreneurs devoted high effort to the project, and decides whether or not to invest.

The payoffs are as follows. Each entrepreneur gets a payment of 5 if the venture capitalist invests and 0 otherwise. In addition, choosing high effort costs an entrepreneur 1, while choosing low effort is free. Investing costs the venture capitalist 2, but if he invests he gains 3 for each entrepreneur who chose high effort. If the venture capitalist does not invest, his payoff is 0. Draw the game tree corresponding to this game and find its equilibrium outcome by solving backwards.

See figure at end of answer key.

Problem 6 Consider the game below:

		Man	
		Threaten	Don't threaten
Thief	Steal	30,30	50,35
	Don't steal	40,60	20,20

a. What is the Nash equilibrium if choices are made simultaneously? There are 2 Nash equilibria in pure strategies. Don't steal, Threaten, and Steal, Don't threaten.

b. What is the equilibrium outcome if player 1 chooses first?

You should draw the game tree in answering this question. If the thief chooses first, he chooses to steal, and the man chooses not to threaten.

c. What is the equilibrium outcome if player 2 chooses first?

The man chooses to threaten, and the thief chooses not to steal.

Problem 7 Two bills are being considered in Congress (bill A, which would reinstitute the Volstead Act, and bill B, which would prohibit anyone of Canadian origin from owning property). Here are the payoffs to Congress and the president depending upon which laws are passed:

Outcome	Congress	President
Bill A only	8	-1
Bill B only	-1	9
Both bills	5	5
Neither bill	0	0

a. Suppose that Congress first decides which of the four options to select. The president can then either sign or *veto*, in which case no law is passed. Which bills become laws in the equilibrium of this sequential game? Explain, with aid of a diagram.

See the diagram on the last page. In the equilibrium, Congress passes both bills, and the President signs them.

b. Now suppose that the president has a *line-item veto*, so that if Congress passes both bills, he can choose to sign bill A or bill B only. However, he cannot enact laws that Congress does not pass. Which bills become laws in the equilibrium of this game? Explain.

Now, if both bills are sent to the President together, he will veto Bill A, but not Bill B. As such, the payoffs to Congress passing both bills are now $(-1, 9)$. The game is otherwise the same as in part a. In the equilibrium, neither bill becomes law.

c. It is often suggested that giving the president a line-item veto would be a good way to make government work more efficiently, as then he would not have to veto entire bills just because he felt one provision of the bill would make a bad law. In light of this question, what do you think of this suggestion?

A line-item veto would have the direct effect of allowing the President to eliminate parts of legislation he views as harmful, but would have the indirect effect of hindering compromise; under a line-item veto, only bills that both Congress and the President agree on can become law, and it may be difficult to work out a compromise where each side contributes his own favored policies to a bill.

