Unit 2.2: Consumer Theory (Graphical Presentation): Consumer Optimization

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1 Finding the Optimal Bundle

The *optimal bundle* is the affordable bundle that leaves the consumer the best off. If preferences are well-behaved, then the optimal bundle is the bundle on the budget constraint that puts the consumer on the highest possible indifference curve.

Illustrating this on a diagram, figure 1 shows a budget line and several different indifference curves.

In figure 1, bundle A is the optimal bundle. It puts the consumer on the highest possible indifference curve that is still affordable (on the budget line).

Let us contrast this with some other bundles. Although bundle B makes the consumer better off than bundle A (since bundle B is on a higher indifference curve), bundle B is not affordable since it is outside of the budget constraint. Therefore, bundle B cannot be the optimal bundle.

Now, bundle C is affordable since it is on the budget constraint. However, notice that bundle C is on a lower indifference curve than bundle A. Thus, bundle C cannot be the optimal bundle – the consumer can stay within the budget set and be better off by consuming bundle A.

The consumer likes bundle D and bundle A equally well since they are on the same indifference curve. However, bundle D is not affordable, so it cannot be the optimal bundle.

2 Characterizing the Optimal Bundle

Notice from figure 1 that at the optimal bundle the indifference curve is just tangent to the budget line. At bundle C, the indifference curve crosses the budget line. This cannot be the optimal bundle since it is possible for the consumer to be on a higher indifference curve while staying on the budget line. That is, the highest indifference curve just touches the budget line.

What this means is that, at the optimal bundle, the slope of the budget line is *equal* to the slope of the indifference curve. Geometrically, this is true any time two curves are tangent. Recall that the slope of the budget line is the (negative of) the price ratio $\frac{P_X}{P_Y}$. The slope of the indifference curve is the (negative of) the marginal rate of substitution, MRS. We can say, then, that *at the optimal bundle*.

$$MRS = \frac{P_X}{P_Y}$$

This condition makes good economic sense. The MRS is the amount of Y that the consumer is *willing* to give up to get one more unit of X. The price ratio is the amount of Y that the consumer *needs* to give up to get one more unit of X, given the prices.

Say that the MRS is equal to 5 but that the price ratio is equal to 2. This means that the consumer is willing to give up 5 apples (good Y) to obtain another fish (good X). What the price ratio means is that, given the prices, the consumer only would need to give up 2 apples to obtain another fish. Clearly, then, this

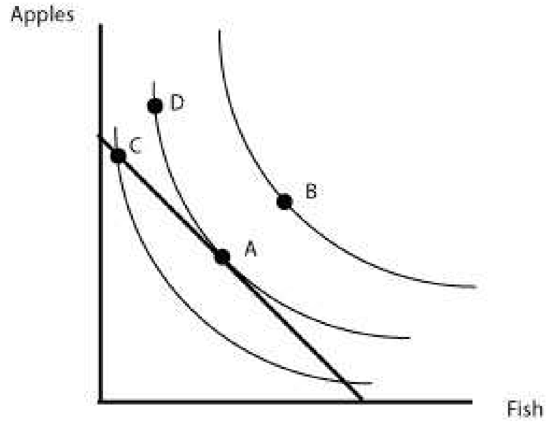

Figure 1: Locating the Optimal Bundle

cannot be the optimal bundle. The consumer could raise his happiness by giving up apples and obtaining more fish. If he would be willing to give up 5 apples to get another fish, but he only needs to give up 2 apples, then clearly he should buy more fish.

Notice that this is true, for example, at bundle C in figure 1. The MRS (slope of the indifference curve) is greater than the price ratio (slope of the budget line. Indeed, the consumer can find a higher indifference curve by moving right along the budget line – buying more fish and giving up apples.

This consumer would get to the optimal bundle by buying more fish. As he does this, the MRS will fall. In this case, he should continue giving up apples and buying more fish until the MRS falls to 2, so that it is equal to the price ratio. This characterizes the optimal bundle.

Exactly the opposite argument applies if the MRS is lower than the price ratio. If the price ratio is 7, but the MRS is 3, then the consumer was only willing to give up 3 apples to obtain another fish, but prices are such that he had to give up 7 apples to obtain his last fish. Clearly, in this case, he should consume *fewer* fish (and more apples).

Combining, the optimal bundle, that makes the consumer as well-off as possible, occurs at the point on the budget constraint where the marginal rate of substitution is exactly equal to the price ratio.

3 Marginal Utility Characterization

Utility is a numerical measure of happiness. In the next unit we will discuss in a deeper sense what, exactly, this means. In essence, the number itself has no real meaning except that higher utility makes the consumer better off than lower utility.

Marginal utility, then, is the increase in utility that results from one more unit of the good. So MU_X is the additional utility that results when the consumer obtains one more unit of good X. This gives us a more precise definition of the marginal rate of substitution.

$$MRS = \frac{MU_X}{MU_Y}$$

This definition makes sense. Suppose that an additional unit of good X generates $MU_X = 10$ additional units of utility, and that an additional unit of good Y generates $MU_Y = 5$ additional units of utility. This means that the consumer needs 2 units of Y to give him the same extra utility as 1 unit of X. But, of course, this is exactly the marginal rate of substitution that we defined earlier – the number of units of Y that the consumer would give up in order to obtain one more unit of X.

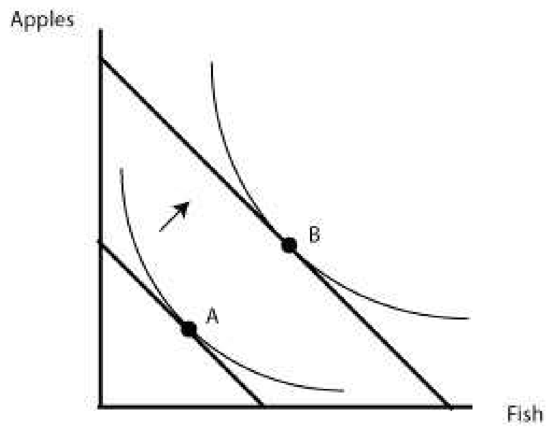

Figure 2: An increase in income

Recall that the optimal bundle was characterized as the bundle where $MRS = \frac{P_X}{P_Y}$. Substituting in our new definition of the MRS gives us another characterization of the optimal bundle:

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

Or, arranged a little bit differently, we could write this condition as:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

This characterization of the optimal bundle is even more intuitive. We can interpret $\frac{MU_X}{P_X}$ as the additional utility obtained *per dollar* spent on the good. For example if the marginal utility of fish is $MU_X = 6$ and the price of a fish is $P_X = 2$, then each of the dollars that is spent on a fish yields 3 extra units of utility.

What this condition says is that, at the optimal bundle, the marginal utility per dollar must be equal across goods. This is very straightforward to understand economically. If each dollar of expenditure on good X generates 5 extra units of utility, but each dollar of expenditure on good Y generates only 3 extra units of utility, then clearly the consumer can improve his total utility by taking some money away from good Y and moving it towards good X. As he buys more X, MU_X will fall (and MY_Y will rise as he buys less Y). He should continue to give up X for Y until he attains equality in the ratio $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$.

4 Comparative Statics

When income changes, the optimal bundle changes. An increase in income (a parallel shift of the budget constraint) is shown in figure 2. The original optimal bundle is bundle A, but after the change in income, but optimal bundle is bundle B.

A price change also causes a change in the optimal bundle. An increase in the price of fish (the good on the X axis) is shown in figure 3. Recall that the budget line pivots inwards along the axis measuring fish. Again, the optimal bundle changes from bundle A to bundle B.

We will study the decomposition of these changes in detail in the next unit.

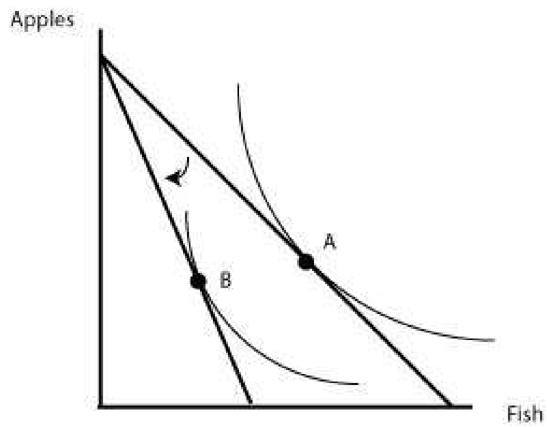

Figure 3: An increase in the price of X

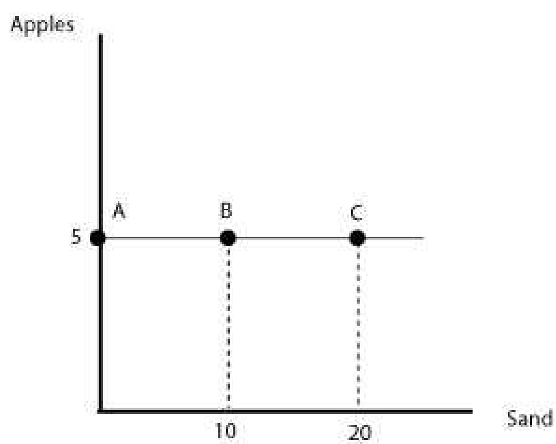

Figure 4: Indifference curve for a useless good

5 Useless Goods

Consider a good like sand. The consumer gets no extra utility by having more sand, but does not dislike having it either. To draw indifference curves, we need to find combinations of the goods that leave the consumer equally well off. Let us start with the bundle containing 5 apples and no sand. Since the consumer does not care about sand, he will like equally well a bundle containing 5 apples and 10 sand, or a bundle containing 5 apples and 20 sand. All these bundles leave him equally well-off. This indifference curve is shown in figure 4. The consumer likes any bundle with 5 apples equally well, irrespective of the amount of sand.

Clearly, bundles with more apples are better than bundles with fewer apples. That is, higher indifference curves are better than lower ones. We use arrows on the indifference maps to indicate the direction of increasing preference. See figure 5.

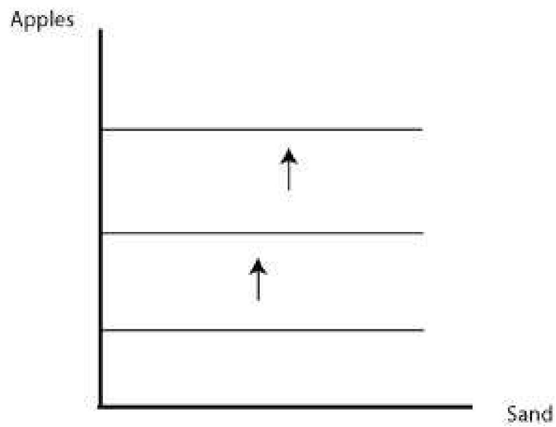

Figure 5: Indifference map for a useless good

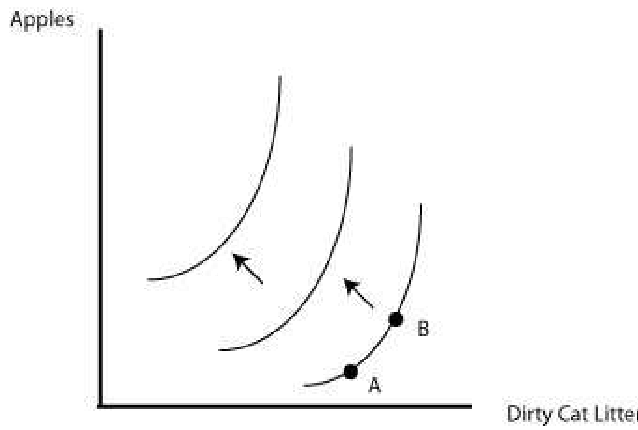

Figure 6: Indifference map for a garbage good

6 Garbage Goods

Now consider a good that the consumer actively dislikes – dirty cat litter, for example. Unlike sand, which the consumer doesn't care about, the consumer dislikes dirty cat litter, and so having more makes him worse off. In this case, if we start with bundle A in figure 6, if we want to give the consumer more dirty cat litter and leave him equally well-off, then we would need to give him *more* apples to compensate him. That is, the indifference curves are upward sloping in this case.

Looking at the entire indifference map in figure 6, indifference curves to the northwest leave the consumer better off – more apples and less cat litter.

7 Perfect Substitutes

Consider a consumer who views Coke and Pepsi to be perfect substitutes. One can of Coke is just as good as one can of Pepsi no matter how much Coke or Pepsi he has. The bundle with 5 Coke and 0 Pepsi leaves the consumer just as well off as the bundle with 0 Coke and 5 Pepsi. These are both just as good as the bundle containing 3 Coke and 2 Pepsi, for example. Notice that the indifference curve is just a straight line

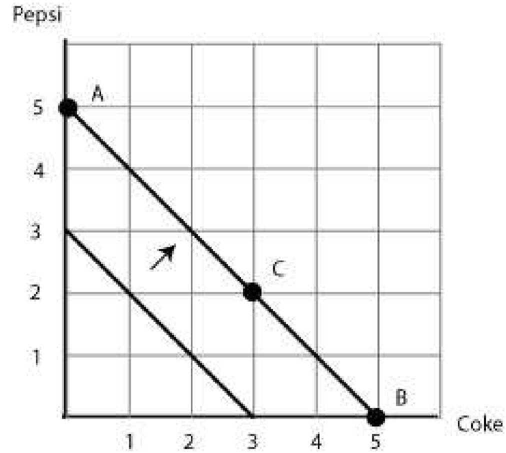

Figure 7: Indifference map for perfect substitutes

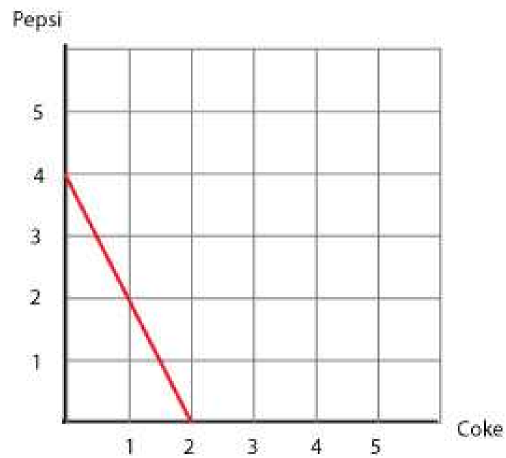

Figure 8: Coke/Pepsi budget line

connecting these bundles. As usual, higher indifference curves with more Coke and Pepsi are preferred to lower indifference curves. See figure 7 for the indifference map.

Now suppose that a can of Pepsi costs \$2.50, a can of Coke costs \$5.00 and that the consumer has an income of \$10. The budget constraint is shown in figure 8.

The optimal bundle is the bundle on the budget constraint that puts the consumer on the highest possible indifference curve. In this case, the consumer is clearly on his highest indifference curve by using his \$10 to consume 4 Pepsi and no Coke. See the indifference map in figure 9.

This makes good intuitive sense. If two goods are perfect substitutes, and one is cheaper than the other, you should buy only the cheaper one and none of the more expensive one.

If the prices had been exactly the same, then the highest indifference curve would overlap the budget line so that the consumer would have been indifferent over any bundle on the budget line. This makes sense too. If two goods are the same price and a consumer considers them to be perfect substitutes, then it really doesn't matter how much of each good the consumer chooses to buy.

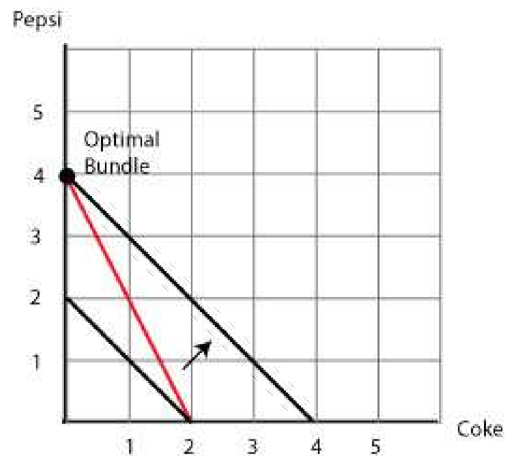

Figure 9: Coke/Pepsi indifference map

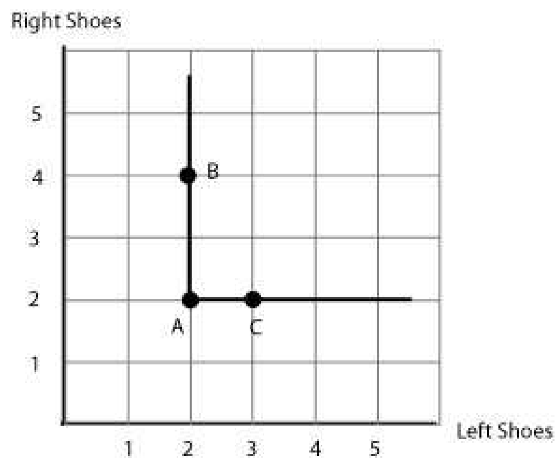

Figure 10: Indifference curve for perfect complements

8 Perfect Complements

Perfect complements are goods that need to be consumed in some fixed ratio. A simple example is left and right shoes, which must be consumed in a 1-for-1 ratio. A left shoe is totally useless without a right shoe, and vice versa.

Consider a bundle containing 2 left shoes and 2 right shoes (bundle A in figure 10). The consumer likes bundle B with 2 left shoes and 4 right shoes equally well since the additional right shoes are useless. Similarly, the consumer likes the bundle with 3 left shoes and 2 right shoes just as well since the additional left shoe is useless. The indifference curve is a right angle. Any bundle on this indifference curve makes the consumer equally happy.

A bundle with 3 left shoes and 3 right shoes would be on a higher indifference curve and would leave the consumer better off than the indifference curve with the bundle containing 2 left shoes and 2 right shoes. As usual, utility increases on the indifference map moving upwards and to the right. See figure 11.

If we combine these indifference curves with the budget constraint, it is fairly obvious that the optimal

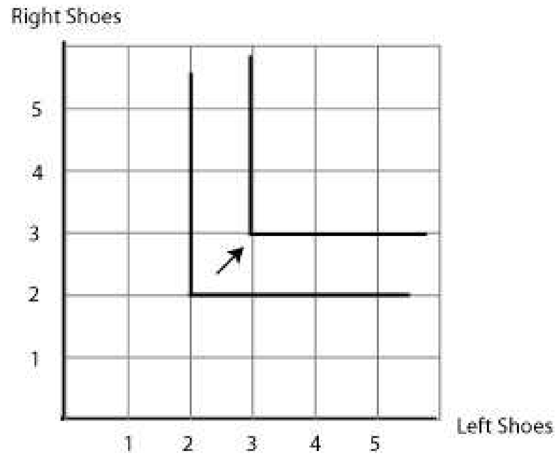

Figure 11: Indifference map for perfect complements

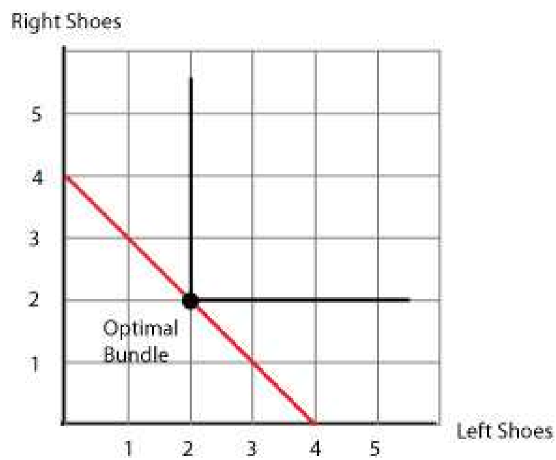

Figure 12: Optimal bundle for perfect complements

bundle is going to occur on a corner point of the indifference curve. Intuitively, unless left and right shoes are bought together, then some of the consumer's money is wasted. For example, in figure 12, when the consumer has \$100 of income, and left and right shoes each cost \$25, then the optimal bundle contains 2 left shoes and 2 right shoes.