Unit 7.1: Game Theory Basics

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1 Motivation

Game theory is the study of strategic decision-making. Strategic decisions are those in which there is interdependence among the agents involved – decisions that one agent makes affect other agents and vice versa. This means that players must consider the responses of other players when making decisions.

More specifically, game theory allows for the possibility that your payoffs depend on the choices that other players make and other players' payoffs depend on the choices that you make. This seems like a basic idea, but it is a surprisingly new line of inquiry in economics.

Competitive markets, with large numbers of agents, do not require game theoretic analysis. Each buyer and seller in a competitive market is so small that he can effectively ignore the feedback effects of his own decisions on the rest of the market. One buyer or seller is not enough to influence the market.

Similarly, decisions involving one agent also do not require game theory. A consumer who maximizes his utility subject to a fixed budget line or a monopoly choosing the price that maximizes profit is not involved in any interactive decision-making.

This leaves the intermediate case – what about settings with more than one agent, but not so many that each player can ignore the effects of his own decisions on the market as a whole? Indeed, oligopolies were the first setting in which economists applied game theoretic reasoning. The price that Airbus sets for selling airplanes is going to affect the profit that Boeing earns, and vice versa. This sort of strategic reasoning is useful in many other settings too. Here are just a few examples.

- Auctions – your best bid for an item in an auction depends on what you think other players will bid
- Contributions to public goods – your optimal contribution to a public good or a charity depends on how much you expect others to donate
- Workplace interactions – the incentive structure that your boss sets up influences how hard you work
- Wars – the optimal strategy for each party in a war surely depends on the reaction expected from the other side
- Household economics – whether you clean the dirty dishes in the sink depends on when (or whether) you expect your spouse to do it if you don't.

2 Setup of a Game

A description of a game includes a few things.

- A set of **players**, describing who plays the game. In the case of an oligopoly price war, the players are the rival firms.

- A set of **strategies**, describing the choices available to each player. In the case of the oligopoly, each firm's strategies are the possible prices it can charge. For an auction, the participants' strategies are their bids.
- A set of **payoffs**, describing the payoff to each player for all combinations of strategies. Any player's payoff can potentially be affected not only by his own strategy, but by the strategies chosen by other players.
- **Rationality**, meaning that each player's objective is to maximize his own payoff. The key point is that the payoff numbers need to represent *everything* about a player's preferences. If a player dislikes a certain strategy, this should be reflected in the payoff number. We assume that players always aim to maximize their payoffs.
- **Common knowledge of the rules** – This is not to say that information cannot be incomplete or asymmetric. Indeed, game theory handles imperfect information quite well. However, all players must understand the basic setup (e.g. strategies and order of moves) of the game.

Given a game, the goal is to locate an *equilibrium*, which is a prediction about the outcome of the game.

3 Sequential and Simultaneous Games

In a *sequential* game, players choose their strategies in some sequence. For example, chess is a sequential game where each player who moves observes all prior moves.

In a *simultaneous* game, players choose their strategies simultaneously. The key is that each player chooses his strategy without knowing what strategy is chosen by other players. A kicker in football who decides where to kick the ball and a goalie deciding where to block are involved in a simultaneous game.

4 One-Shot and Repeated Games

In a *one-shot* game, the specified interaction between a given set of players occurs one time.

In a *repeated* game, the same players repeat the game multiple times. Obviously, repeated games can provide different incentives than one-shot games. Cooperation is more likely in a repeated game. In particular, defection against a cooperative agreement is more likely in a one-shot game since there is no future consequence for doing so.

5 Pure Strategies and Mixed Strategies

A player plays a *pure strategy* if he plays a particular one of his strategies with probability 1.

A player plays a *mixed strategy* if he deliberately randomizes among different strategies.

6 Discrete Strategies and Continuous Strategies

A *discrete* strategy set means that a player picks from a finite number of choices. For example, in the housework game, "wash the dishes" and "not wash the dishes" are the two strategies available to each spouse.

A *continuous* strategy set means that a player picks from a continuum of choices. For example, in the oligopoly rivalry, the firms can set any price. Prices are not restricted to a discrete set.

Methodologically, maximization among discrete strategies involves inspecting the various choices. Maximization over a continuous strategy set involves the use of calculus.

7 Information Structure

Information in a game may be *asymmetric*, meaning that players have different information. For example, your mechanic knows more about what is wrong with your car than you do. Further, information may be *incomplete*, meaning that there is general uncertainty about the setting. For example, all airlines setting ticket prices to ski resorts are uncertain about what the weather is going to be.

8 Cooperative and Noncooperative Games

In *cooperative games*, players can create enforceable agreements before the game begins such that it is impossible for players to deviate from these agreements. Essentially, this allows coalitions to form in advance for players to act in their mutual best interest.

In *noncooperative games*, players are free to choose any strategy they want. Any agreements have to be self-enforceable (meaning that players find it in their own best interest to go along with the agreement) since noncooperative games give players the freedom to choose any of their strategies.

Unit 7.2: Simultaneous Games

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Simultaneous games are those where players choose their strategies simultaneously. That is, each player must choose his strategy without observing the strategies selected by other players.

1 Nash Equilibrium

A *best response* is the strategy that gives a player his highest possible payoff *given* the strategies chosen by other players.

The basic solution concept for simultaneous games is Nash Equilibrium. A *Nash Equilibrium* is a set of strategies such that each player in the game chooses a best response given the strategies selected by other players. Stated differently, a set of strategies is a Nash Equilibrium if no player could switch strategies and obtain a higher payoff, given the strategies selected by other players.

2 Two-Player Games with Discrete Strategy Sets

Consider a game where player 1 can select from three strategies: Top, Middle and Bottom. Player 2 can select from two strategies: Left and Right. Two-player simultaneous games with discrete strategies can be represented in a table. By convention, player 1's strategies are listed in rows and player 2's strategies are listed in columns.

	Left	**Right**
Top	3,3	3,4
Middle	5,5	2,6
Bottom	4,6	1,4

Notice that there are six possible outcomes in this game. The numbers in each cell show the payoff to player 1 and the payoff to player 2, respectively, when a particular set of strategies is chosen.

Player 1's best responses depend on the strategies chosen by player 2:

- If player 2 plays Left, player 1's best response is Middle (since the choices are 3 from Top, 5 from Middle and 4 from Bottom)
- If player 2 plays Right, player 1's best response is Top (since the choices are 3 from Top, 2 from Middle and 1 from Bottom)

Player 2's best responses depend on the strategies chosen by player 1:

- If player 1 plays Top, player 2's best response is Right (since the choices are 3 from Left and 4 from Right)
- If player 1 plays Middle, player 2's best response is Right (since the choices are 5 from Left and 6 from Right)

- If player 1 plays Bottom, player 2's best response is Left (since the choices are 6 from Left and 4 from Right)

Now, indicate these best responses on the game matrix.

	Left	**Right**
Top	3,3	**3**,**4**
Middle	**5**,5	2,**6**
Bottom	4,**6**	1,4

The Nash equilibrium of this game is (Top, Right) – where both players are playing a best response. To see what this means, neither player wants to deviate given what the other player is doing. When player 2 selects right, player 1 does not want to deviate away from Top. Similarly, when player 1 selects top, player 2 does not want to deviate away from Right. This set of strategies is a mutual best response. No player wants to unilaterally deviate when the other player follows his equilibrium strategy.

Notice that (Bottom, Left) gives both players a higher payoff, but it is not a Nash Equilibrium. If player 2 selects Left, then player 1's best response is Middle. Similarly, (Middle, Left) is not a Nash Equilibrium. If player 1 selects Middle – player 2's best response is to select Right, not Left. (Top, Right) is the only set of strategies where the two players are simultaneously playing a best response.

For another example, here is probably the most well-known game in game theory.

	C	**D**
C	2,2	-1,3
D	3,-1	0,0

For player 1's best responses:

- If player 2 plays C, player 1's best response is D (3 vs. 2).
- If player 2 plays D, player 1's best response is D (0 vs. -1).

For player 2's best responses:

- If player 1 plays C, player 2's best response is D (3 vs. 2).
- If player 1 plays D, player 2's best response is D (0 vs. -1).

Indicating the best responses on the table.

	C	**D**
C	2,2	-1,**3**
D	**3**,-1	**0**,**0**

The Nash Equilibrium is (D,D). Notice that (C,C) is better for both players, but this is not an equilibrium since – if player 2 selected C, player 1's best response is to play D, not C (the same can be said of player 2).

Any game with this structure is known as a *prisoner's dilemma*. These appear all over the place in economics, and in other disciplines as well. For example, two firms might make more profit if they agree to charge a high price than if both charge a low price. However, this is not a Nash Equilibrium – if firm A charges a high price, then firm B can make more profit by charging a lower price and stealing all of firm A's customers. Ultimately, the equilibrium is where both charge a low price and end up worse off than if they had maintained their agreement.

A game can have multiple Nash Equilibria. Consider the following game.

	a	**b**	**c**	**d**	**e**
A	1,4	0,6	5,2	2,3	2,5
B	2,2	3,1	3,1	4,3	3,3
C	7,3	3,5	0,0	3,2	0,0

We can find the best responses as above. There is one subtlety here. Notice that, when player 2 plays b, player 1's best response is B or C, since both give him a payoff of 3. Similarly, when player 1 plays B, player 2's best response is d or e.

	a	**b**	**c**	**d**	**e**
A	1,4	0,**6**	**5**,2	2,3	2,5
B	2,2	**3**,1	3,1	**4**,**3**	**3**,**3**
C	**7**,3	**3**,**5**	0,0	3,2	0,0

This game has three Nash Equilibria: (B,d), (B,e) and (C,b).

As a final example, consider the game below.

	L	**C**	**R**
T	7,1	4,7	2,4
M	5,-2	5,4	1,0
B	2,5	8,3	2,4

Indicating the best responses:

	L	**C**	**R**
T	**7**,1	4,**7**	**2**,4
M	5,-2	5,**4**	1,0
B	2,**5**	**8**,3	**2**,4

This game has no Nash Equilibrium (at least in pure strategies). There is no set of strategies where both players play a mutual best response to each other.

3 Dominant Strategies

Consider the following game.

	Y	**Z**
A	6,1	2,4
B	7,2	5,3
C	1,2	3,6

For player 1's best responses:

- If player 2 plays Y, player 1's best response is B
- If player 2 plays Z, player 1's best response is B

Notice that player 1 should always play B no matter which strategy player 2 selects. In this case, we say that strategy B is a *dominant strategy* for player 1. In general, a player has a dominant strategy if that strategy is always a best response regardless of what strategies are chosen by other players.

Similarly, for player 2's best responses:

- If player 1 plays A, player 2's best response is Z

- If player 1 plays B, player 2's best response is Z
- If player 1 plays C, player 2's best response is Z

So Z is a dominant strategy for player 2.

For this game, we say that (B,Z) is a *dominant strategy equilibrium*, since all players are using a dominant strategy. Notice that a dominant strategy equilibrium is certainly a Nash Equilibrium, although a Nash equilibrium does not need to be a dominant strategy equilibrium. If a game has a dominant strategy equilibrium, this is a very convincing equilibrium outcome. If a particular strategy gives a player his highest possible payoff *regardless* of the strategies selected by his opponents, then it is fairly clear that the player will play this strategy.

Here is another game:

	Y	**Z**
A	5,4	4,5
B	7,3	3,7

Indicating the best responses below:

	Y	**Z**
A	5,4	**4**,**5**
B	**7**,3	3,**7**

The Nash Equilibrium is (A,Z). Notice that player 2 has a dominant strategy. When 1 selects A, player 2's best response is Z. Similarly, when 1 selects B, player 2's best response is Z. Player 2 should pick Z regardless of what player 1 selects.

However, player 1 does not have a dominant strategy. If 2 selects Y, player 1's best response is B. On the other hand, if player 2 selects Z, player 1's best response is A. As a result (A,Z) is not a dominant strategy equilibrium, since a dominant strategy equilibrium requires that *all* players be using a dominant strategy.

There is another important class of games known as *coordination games*. Here is an example. Two teammates are writing a computer program, and can write it in C++ or in Java.

	C++	**Java**
C++	1,1	0,0
Java	0,0	1,1

Both (C++,C++) and (Java,Java) are obviously equilibria. It doesn't matter which one they coordinate on, as long as they both agree.

Consider now the following game, where the builders of a car can choose to use Metric or British measurements.

	Metric	**British**
Metric	2,2	0,0
British	0,0	1,1

While (Metric,Metric) is a Nash Equilibrium, and is clearly the Pareto superior outcome, (British,British) is also a Nash Equilibrium. Furthermore, Metric is not even a dominant strategy for either player. If player 2 selects British, then player 1's best response is also British. For metric to be a dominant strategy, it would need to be a best response even if player 2 picked British.

4 Games with Continuous Strategy Sets

Suppose that player 1 chooses a_1 and that player 2 chooses a_2. Unlike in the previous games, a_1 and a_2 can be any number. The strategy choices are not limited to any discrete set. The player's payoffs are, respectively:

$$\Pi_1 = 3a_1 - a_1 a_2 - a_1^2$$
$$\Pi_2 = 4a_2 - a_1 a_2 - a_2^2$$

Notice that this is game theoretic because player 1's payoff Π_1 depends on player 2's strategy choice a_2 and vice versa.

In a Nash Equilibrium, player 1 plays a best response given the strategy of player 2. In other words, player 1 picks the a_1 that maximizes his own payoff given player 2's choice of a_2. As usual, we find this maximizer using calculus.

$$\frac{\partial \Pi_1}{\partial a_1} = 3 - a_2 - 2a_1 = 0$$
$$a_1 = \frac{3 - a_2}{2} = \frac{3}{2} - \frac{1}{2}a_2$$

This is called player 1's *best-response function* or his *reaction function.* It describes player 1's best strategy for any particular choice of a_2. In other words, the a_1 that maximizes player 1's payoff conditional on a_2.

Now, player 2 picks a_2 to maximize his own payoff.

$$\frac{\partial \Pi_2}{\partial a_2} = 4 - a_1 - 2a_2 = 0$$
$$a_2 = \frac{4 - a_1}{2} = 2 - \frac{1}{2}a_1$$

This is player 2's best response to any particular choice of a_1.

A Nash Equilibrium occurs where both players play a best response. So we need to solve the following system of equations:

$$a_1 = \frac{3}{2} - \frac{1}{2}a_2$$
$$a_2 = 2 - \frac{1}{2}a_1$$

Substituting the second equation into the first:

$$a_1 = \frac{3}{2} - \frac{1}{2}\left(2 - \frac{1}{2}a_1\right)$$
$$a_1 = \frac{3}{2} - 1 + \frac{1}{4}a_1$$
$$\frac{3}{4}a_1 = \frac{1}{2}$$
$$a_1 = \frac{2}{3}$$

Substituting back gives:

$$\begin{aligned} a_2 &= 2 - \frac{1}{2}a_1 \\ &= 2 - \frac{1}{2}\left(\frac{2}{3}\right) = \frac{5}{3} \end{aligned}$$

Summarizing, the Nash Equilibrium of this game is $a_1 = \frac{2}{3}$ and $a_2 = \frac{5}{3}$. This is the set of strategies where players are mutually picking their best responses given the strategy of the other player.

5 Cournot Oligopoly

Consider an oligopoly setting with two firms. The market demand curve is

$$P = 140 - 2Q$$

where P is the market price and Q is the total market output. Q is the sum of the output by each firm. $Q = q_1 + q_2$. Firm 1 chooses q_1 and firm 2 chooses q_2. The marginal cost of each unit of output for either firm is 20.

Let us write out an expression for the profit that firm 1 earns:

$$\begin{aligned} \Pi_1 &= TR_1 - TC_1 \\ &= P \cdot q_1 - 20q_1 \\ &= (140 - 2Q)q_1 - 20q_1 \\ &= (140 - 2(q_1 + q_2))q_1 - 20q_1 \\ &= (140 - 2q_1 - 2q_2)q_1 - 20q_1 \\ &= 140q_1 - 2q_1^2 - 2q_1q_2 - 20q_1 \\ &= 120q_1 - 2q_1^2 - 2q_1q_2 \end{aligned}$$

Notice that this is game theoretic because player 2's output q_2 affects the profit that firm 1 can earn (via the market price – firm 2 raising output lowers the price at which firm 1 can sell its output). Writing a similar expression for firm 2's profit:

$$\begin{aligned} \Pi_2 &= TR_2 - TC_2 \\ &= P \cdot q_2 - 20q_2 \\ &= (140 - 2Q)q_2 - 20q_2 \\ &= (140 - 2q_1 - 2q_2)q_2 - 20q_2 \\ &= 120q_2 - 2q_2^2 - 2q_1q_2 \end{aligned}$$

In the Nash Equilibrium (for this particular game, the Nash Equilibrium outcome is called the *Cournot Equilibrium*), both firms pick output to maximize their profits. In other words, both firms play best responses.

Firm 1 picks q_1 to maximize profit:

$$\frac{\partial \Pi_1}{\partial q_1} = 120 - 4q_1 - 2q_2 = 0$$

$$\begin{aligned} 4q_1 &= 120 - 2q_2 \\ q_1 &= 30 - \frac{1}{2}q_2 \end{aligned}$$

This is firm 1's best response function. The formula gives the profit-maximizing level of output for firm 1 to produce depending upon the output by firm 2 q_2.

Similarly, we find firm 2's best response function by picking q_2 to maximize his profit:

$$\begin{aligned}\frac{\partial \Pi_2}{\partial q_2} = 120 - 4q_2 - 2q_1 = 0 \\ 4q_2 = 120 - 2q_1 \\ q_2 = 30 - \frac{1}{2}q_1\end{aligned}$$

The Nash Equilibrium occurs where both players play a best response to each other. In other words, we find the q_1 and q_2 that simultaneously solve both firms' reaction functions. Substituting firm 2's reaction function into firm 1's:

$$\begin{aligned}q_1 &= 30 - \frac{1}{2}q_2 \\ q_1 &= 30 - \frac{1}{2}\left(30 - \frac{1}{2}q_1\right) \\ q_1 &= 30 - 15 + \frac{1}{4}q_1 \\ \frac{3}{4}q_1 &= 15 \\ q_1 &= 20\end{aligned}$$

Substituting back for q_2:

$$q_2 = 30 - \frac{1}{2}q_1 = 20$$

The Cournot equilibrium outcome in this market is $q_1 = 20$ and $q_2 = 20$, meaning that $Q = 40$ and $P = 140 - 2Q = 60$. In this case, equilibrium profits are:

$$\begin{aligned}\Pi_1 = TR_1 - TC_1 = 20(60) - 20(20) = 800 \\ \Pi_2 = TR_2 - TC_2 = 20(60) - 20(20) = 800\end{aligned}$$

One final thing to observe is that this is a prisoner's dilemma. Had the firms produced $q_1 = 15$ and $q_2 = 15$, profits would have been $\Pi_1 = 900$ and $\Pi_2 = 900$. The problem is that this is not an equilibrium. If firm 2 produces $q_2 = 15$, then firm 1's best response would be $q_1 = 30 - \frac{1}{2}q_2 = 22.5 \neq 15$.

6 Mixed Strategies

Consider the following game, played between a professor and a student. The student (player 1) can cheat or be honest. The professor can monitor or relax.

	Monitor	**Relax**
Cheat	-5,10	5,-5
Honest	2,-1	2,0

This game has no Nash Equilibrium in pure strategies. However, suppose that instead of picking pure strategies, players picked randomized strategies. That is, player 1's strategy is to cheat with some probability and be honest with some probability.

Let p_{cheat} and p_{honest} be the probabilities with which player 1 cheats and is honest, respectively. Similarly, let $p_{monitor}$ and p_{relax} be the probabilities with which player 2 monitors and relaxes, respectively.

When player 2 randomizes, player 1's expected payoffs from cheating and being honest are:

$$E\Pi_{cheat} = -5p_{monitor} + 5p_{relax}$$
$$E\Pi_{honest} = 2p_{monitor} + 2p_{relax}$$

Here is the basic idea of mixed strategy equilibrium. Player 1 will only randomize between these two strategies if each gives him the same expected payoff. If one of the two strategies gave him a higher payoff, then he would select that one for sure, taking us back to the pure strategy case. For player 1 to randomize between cheating and being honest, both have to give him the same expected payoff:

$$E\Pi_{cheat} = E\Pi_{honest}$$
$$-5p_{monitor} + 5p_{relax} = 2p_{monitor} + 2p_{relax}$$

To solve this, observe that player 2 either relaxes or monitors, so $p_{relax} = 1 - p_{monitor}$.

$$-5p_{monitor} + 5(1 - p_{monitor}) = 2p_{monitor} + 2(1 - p_{monitor})$$
$$-5p_{monitor} + 5 - 5p_{monitor} = 2p_{monitor} + 2 - 2p_{monitor}$$
$$3 = 10p_{monitor}$$
$$p_{monitor} = \frac{3}{10}$$

In words, when the professor monitors with probability $\frac{3}{10}$ and relaxes with probability $\frac{7}{10}$, then the student is indifferent between cheating and being honest, and so he is willing to randomize. His expected payoff is the same regardless of whether he cheats or is honest

Similarly, player 2 must be willing to randomize. The expected payoffs from his two strategies are

$$E\Pi_{monitor} = 10p_{cheat} + -1p_{honest}$$
$$E\Pi_{relax} = -5p_{cheat} + 0p_{honest}$$

Player 2 will play a mixed strategy only when he is indifferent between his two strategies:

$$E\Pi_{monitor} = E\Pi_{relax}$$
$$10p_{cheat} + -1p_{honest} = -5p_{cheat} + 0p_{honest}$$

Since $p_{honest} = 1 - p_{cheat}$:

$$10p_{cheat} + -1(1 - p_{cheat}) = -5p_{cheat} + 0(1 - p_{cheat})$$
$$10p_{cheat} - 1 + p_{cheat} = -5p_{cheat}$$
$$16p_{cheat} = 1$$
$$p_{cheat} = \frac{1}{16}$$

In words, when the student cheats $\frac{1}{16}$ of the time and is honest $\frac{15}{16}$ of the time, then the professor is indifferent between monitoring the exam and relaxing.

Summarizing, the mixed-strategy Nash Equilibrium is for the student to cheat with probability $\frac{1}{16}$ and to be honest with probability $\frac{15}{16}$. The professor monitors with probability $\frac{3}{10}$ and relaxes with probability $\frac{7}{10}$. We can formally state the Nash Equilibrium as:

$$\left(\frac{1}{16}Cheat + \frac{15}{16}Honest, \frac{3}{10}Monitor + \frac{7}{10}Relax\right)$$

Again, the critical point about this equilibrium is that player 1 – with these probabilities – is indifferent between cheating and being honest, and player 2 is indifferent between monitoring and relaxing. This is the only way to induce players to play a mixed strategy. Notice that equating player 1's payoffs determines the equilibrium mixing probabilities of *player 2* and vice versa.

As a final example, consider the following game:

	L	**R**
T	0,0	-1,5
B	5,-1	-10,-10

Notice that (T,R) and (B,L) are pure strategy Nash Equilibria of this game. There is also a mixed strategy equilibrium.

In order for player 1 to mix between T and B, we need:

$$\begin{aligned} E\Pi_T &= E\Pi_B \\ 0p_L + -1p_R &= 5p_L + -10p_R \\ 0p_L + -1(1-p_L) &= 5p_L + -10(1-p_L) \\ -1 + p_L &= 5p_L - 10 + 10p_L \\ 9 = 14p_L &\Rightarrow p_L = \frac{9}{14} \end{aligned}$$

This implies that $p_R = \frac{5}{14}$ in the mixed strategy equilibrium. Similarly, for player 2 to be willing to play a mixed strategy, we need:

$$\begin{aligned} E\Pi_L &= E\Pi_R \\ 0p_T + -1p_B &= 5p_T + -10p_B \\ 0p_T + -1(1-p_T) &= 5p_T + -10(1-p_T) \\ -1 + p_T &= 5p_T - 10 + 10p_T \\ 9 = 14p_T &\Rightarrow p_T = \frac{9}{14} \end{aligned}$$

This implies that $p_B = \frac{5}{14}$. This game has three Nash Equilibria: the two pure strategy equilibria discussed earlier and the mixed strategy equilibrium that we just solved for. Formally, the set of Nash Equilibria is:

$$\left[(T,R),(B,L),\left(\frac{9}{14}T + \frac{5}{14}B, \frac{9}{14}L + \frac{5}{14}L\right)\right]$$

A few general points:

- Games with a finite number of players and a finite strategy set for all players always have at least one Nash Equilibrium, possibly in mixed strategies. This was John Nash's famous result. There are conditions guaranteeing the existence of a Nash Equilibrium in games with continuous strategy sets as well, though these are more technical. Problems can arise if the number of players is infinite.
- The number of Nash Equilibria, if finite, is in general odd.
- In two-player games where each player has two strategies, there is a mixed equilibrium as long as there aren't any dominant strategies.
- In games with more than two strategies, things can get more complicated because equilibria can be *completely mixed* (where players randomize over all their strategies) or *partially mixed* (where players randomize over some of their strategies, but not all). While solving for the mixed equilibria numerically gets cumbersome, the basic logic remains the same. Players only mix between strategies when they are indifferent between them; in other words, when their expected payoff for those strategies is equal.