## Problem set 3

"due" 3/4/2010

**Problem 1** The Grand Theater is a movie house in a medium-sized college town. On any given night, if the theater is open, it must pay \$500 in fixed costs (paying electricity, ushers, etc) regardless of how many people come to the theater. If the theater is closed, its costs are 0. There are two groups of people who come to the Grand Theater, students and non-students. Students have demand function  $q_s = 220 - 40p_s$  while non-students have demand function  $q_n = 140 - 20p_n$ .

- a. Suppose that the theater cannot tell students apart from non-students. What price will it charge? How many students will come? How many non-students? What will the profits of the Grand Theater be?
- b. Now suppose that the cashier can accurately tell students from non-students by asking students to show their student IDs. Students cannot resell their tickets to non-students after purchase. Will the Grand charge students and non-students different prices? What will these prices be? What will be the Grand's profits?
- c. Finally, suppose that the Grand Theater can only hold 150 people. If the theater is able to charge separate prices to students and non-students, what prices will it charge, and how many students and non-students will come?

**Problem 2** Your firm produces 2 products, each at 0 marginal cost. You face four types of customers, each comprising 25% of your total customers (say you have N total customers). The groups have the following willingness to pay for your product:

customer	${\rm good}\ 1$	good 2
A	\$25	\$100
B	\$40	\$80
C	\$80	\$40
D	\$100	\$25

- a. Compare selling these two products separately to bundling them and selling them together for one price. Which leads to a higher profit?
- b. Now consider the possibility that you sell these goods both bundled and unbundled (that is, you set three prices, one for good 1 alone, one for good 2 alone, and one for the bundle of good 1 and good 2). Would doing this improve upon the outcome of part a? Explain.
- c. Now suppose that the production of each good entails a marginal cost of \$30. How does this information change your answers to a and b above? Is it better to sell the goods unbundled, bundled, or both bundled and separately?

**Problem 3** Firm 1 and Firm 2, are Cournot competitors. The market demand curve is  $p = 120 - q_1 - q_2$ . Firm 1 has a constant marginal cost of \$20, while Firm 2's is \$10.

- a. What are the Cournot equilibrium quantities? What is the equilibrium price?
- b. How much profit does each firm make in the Cournot equilibrium?

**Problem 4** Consider a market with demand Q = 20 - P supplied by two firms engaged in Bertrand price competition. That is, each firm simultaneously names a price, and then whichever firm names the lower price sells 20 - P units, and the firm naming the higher price sells 0. Assume both firms have a constant marginal cost of production equal to \$5.

- a. Under price competition, what price does firm 1 set in an equilibrium? Firm 2?
- b. How much profit does firm 1 earn? Firm 2?
- c. Suppose now that the two firms form a cartel and act as a monopolist. By how much would they be able to increase total profit? What would happen to price?

**Problem 5** Two Cournot quantity competitors face the following demand curve:

$$P = 9 - q_1 - q_2$$

to keep the problem simple, assume that both firms can produce at zero cost.

- a. Solve for the Nash equilibrium. What are  $q_1$  and  $q_2$  in equilibrium, and how much profit does each firm earn?
- b. Suppose firm 1 has the option of outsourcing 4.5 units of production at cost K, and producing nothing on its own (essentially, it is paying K to commit to producing 4.5 units). Firm 2 is aware of firm 1's outsourcing decision prior to its determining how much to produce itself. What is the maximum value of K for which outsourcing is a good idea for firm 1?

Problem 6 Duopoly quantity-setting firms face the market demand

$$p = 150 - q_1 - q_2$$

Each firm has a marginal cost of \$60/unit.

- a. What is the Cournot equilibrium?
- b. What is the Stackelberg equilibrium when firm 1 moves first?
- c. If firm 1 is currently a Cournot competitor, how much would it be willing to pay to become a Stackelberg leader?