Problem set 5

answers

Problem 1 By next year, the stock you own has a 25% probability of being worth \$400 and a 75% probability of being worth \$200. What are the expected value and the variance?

Expected value is $.25 * 400 + .75 * 200 = 250$. Variance is $.25(400 - 250)^2 + .75(200 - 250)^2 = 7500$.

Problem 2 Lisa just inherited a vineyard from a distant relative. In good years (no rain or frost), she earns \$10,000 from the vineyard. In bad years, she earns only \$2,500. She estimates that the probability of a good year is 60%.

a. Calculate the expected value and variance of Lisa's income from the vineyard.

Expected value is $.6 * 10,000 + .4 * 2,500 = 7,000$. The variance is $.6 * (10,000 - 7,000)^2 + .4 * (2,500 - 7,000)^2 = 13,500,000$.

b. Suppose Lisa has utility function $u(w) = \sqrt{w}$, where w is her wealth. Assume she has 0 initial wealth. Ethan, a grape buyer, offers to lease the vineyard from Lisa for \$6,500 next year, so that Lisa would get \$6,500 regardless of whether it was a good year or a bad year. Will Lisa accept this offer?

Her expected utility from the gamble is $.6\sqrt{10,000} + .4\sqrt{2,500} = 80$. She will accept any amount with certainty that gives her at least 80 utility. Her utility from getting \$6,500 with certainty is $\sqrt{6,500} = 80.62$, so yes, she will accept Ethan's offer.

c. Why might Ethan make such an offer? Give three reasons, and explain each. One of these reasons should refer to his attitude toward risk.

It could be Ethan is naturally less risk averse than Lisa (for example, he may be risk neutral). It could be that he has a much greater wealth than Lisa, and so even if he has the same utility function as Lisa, the sums involved may be much smaller to him, making him less risk averse. Finally, it could be that he can diversify the risk of a bad grape harvest, perhaps by buying other vineyards in other locations or by buying stock in a beer company.

Problem 3 Larry owns a house worth \$100,000. There is a 10% chance it will burn down, in which case it will be worth \$20,000. There is a 90% chance it will not burn down and continue to be worth \$100,000. Larry's utility function is $u(w) = \sqrt{w}$, where w is how much his house is worth.

a. Suppose Eagle Insurance offers Larry \$1 worth of insurance for 10 cents. That is, Larry can transfer wealth to the state of the world in which the house burns down from the state in which it does not at the rate 10:1. How much insurance will Larry purchase?

Larry is risk averse, so if offered actuarially fair insurance (i.e. the insurance company will break even, on average, on this policy), he will fully insure, meaning he will buy \$72,727.27 worth of insurance at a cost of \$7,272.72. Showing this rigorously requires a bit of calculus, we can also just keep track of the fact that a risk averse person buys enough just enough actuarially-fair insurance to equalize his wealth across states of the world, which is true generally.

b. Is Eagle's price for \$1 of insurance likely to be higher or lower than that of part a? Why? Will Larry buy more or less insurance than in part a?

The insurance company will have to charge a higher than break even price to cover overhead, employees, etc. Thus, Larry will less than fully insure, i.e. he will purchase less than \$72,727.27 worth of insurance.

Problem 4 Suppose that two investments have the same three payoffs, but the probabilities associated with each payoff differs, as follows:

payoff	**Probability (investment A)**	**Probability (investment B)**
\$300	.10	.30
\$250	.80	.40
\$200	.10	.30

a. Find the expected return and standard deviation of each investment.

Investment A has expected value $300 * .1 + 250 * .8 + 200 * .1 = 250$, variance $.1 * (300 - 250)^2 + .8 * (250 - 250)^2 + .1 * (200 - 250)^2 = 500$, and standard deviation $\sqrt{500} = 22.36$. Investment B has expected value 250, variance 1500, and standard deviation 38.73.

b. Jill has the utility function $u(w) = 5w$, where w is the investment's payoff (assume she has initial wealth 0). Which investment does she prefer?

Jill's expected utility from both investment A and investment B is 1250. She is indifferent between the two gambles. We can tell this immediately by recognizing she is risk-neutral (linear utility function), meaning she chooses the gamble with the higher expected value.

c. Ken has the utility function $u(w) = 5\sqrt{w}$. Which investment will he choose?

EU(investment A) = 78.98, EU(investment B) = 78.82, so he will choose investment A. We can tell this immediately by recognizing that Ken is risk-averse (concave utility function), and so will choose the gamble with the lower standard deviation, since they have the same expected value.

d. Delores has the utility function $u(w) = 5w^2$. Which will she choose?

Without doing any calculation, she is risk loving (convex utility function) and so will choose the gamble with the higher standard deviation, as they both have the same expected value. thus she will choose investment B.

Problem 5 Suppose that, for a certain model of used car, 25% of owners value their car at \$20,000, 25% at \$15,000, 25% at \$10,000, and 25% at \$5,000. Suppose that buyers value this particular used car at 1.2 times its value to its owner. Owners know how much they value their car, but this is private information; to buyers, all cars look the same.

a. If all four types of used car are traded, what is the highest price a buyer would be willing to pay for a used car? At this price, which owners would be willing to sell?

The highest price a buyer would pay would be \$15,000. At this price, only the bottom three types would be willing to trade.

b. Now suppose the owners with the \$20,000 cars do not participate in the used car market, but all other owners do. Now what is the maximum price a buyer is willing to pay for a used car? Now which sellers are willing to participate in the market?

Now the highest price a buyer is willing to pay is \$12,000. Only the sellers with the two worst types of cars are willing to participate.

c. Argue that adverse selection causes this market to partially unravel, such that only the worst used cars are traded in any equilibrium.

If only the sellers with the two worst types of cars sell, then a buyer is willing to pay only \$9,000, which means the sellers who value their cars at \$10,000 will drop out. If only the worst sellers trade, a buyer is

willing to pay $6,000, and so at any price between $5,000 and $6,000 the worst sellers are willing to sell and buyers are willing to buy.

Problem 6 In *Freakonomics*, economist Steven Levitt discusses the online dating world. According to him, about 40 million Americans a year try to date online. Clearly, there are information problems in online dating, as the person writing the listing has private information about himself/herself. About 70% of online daters list themselves as having "above average" looks, with only about 1% of online daters having "below average looks". Online men claim to be an inch taller than average, and online women claim to be 20 pounds lighter than average. 57% of men and 23% of women posting profiles online never get even one response.

a. Explain why this "market" is not functioning well. What is the economic term we use to describe this problem?

The market is not functioning well because as quality is unobservable, above average people are driven out of the market, lowering the average and driving more people out, etc. This is called adverse selection.

Consider the following simple model of online dating. Suppose 30% of people are of quality 1 (lemons!), 20% are of quality 3, 30% are of quality 5, and 20% are of quality 10, whereh "quality" is some measure of a persons attractiveness to potential dates. Quality is perfectly observable in the real world but completely unobservable in the online world. In the real world (where quality is observable), people of quality 1 can only date other people of quality 1; individuals of quality 3 can date individuals of quality 3 or lower, etc Anyone can find a date matching his/her own quality in the real world if he/she chooses. However, dating online requires much less effort. As such, the utility from going on a date with someone of quality q met in the real world is $u(q) = q$, while the utility from going on a date with someone of quality q met in the online world is $u(q) = 1.5q$.

b. Suppose that all individuals look for a date online, what is the expected utility from going on a date with a person met online?

Expected utility is $.3 * 1.5 * 1 + .2 * 1.5 * 3 + .3 * 1.5 * 5 + .2 * 1.5 * 10 = 6.6$.

c. Considering your answer in b, who will look for a date online and who will look for a date in the real world? Explain briefly.

If individuals believe all types are participating, types 1, 3, and 5 will actually look for a date online.

d. Recompute the expected utility of dating online after considering your result in c. Who looks for a date online now?

If now individuals believe only types 1, 3, and 5 participate, expected utility is $\frac{3}{8} * 1.5 * 1 + \frac{2}{8} * 1.5 * 3 + \frac{3}{8} * 1.5 * 5 = 4.5$. Thus only types 1 and 3 would look for a date online.

e. Again recompute the expected utility of dating online after considering your result in d. Who looks for a date online now?

If only types 1 and 3 are believed to be participating, expected utility is $.6 * 1.5 * 1 + .4 * 1.5 * 3 = 2.7$. Thus only type 1 would look for a date online.

f. Think about the results above Who dates online in the equilibrium of this game? Explain why this is an equilibrium.

If only type 1's are believed to be online, expected utility is 1.5. Given this, type 1's would participate in the market. It is thus an equilibrium for type 1's, and no one else, to participate.

g. Explain why it is not an equilibrium for everyone to date online.

Because online daters cannot differentiate themselves *ex ante*, above average people (type 10's) do better in the real world. If they could select only other type 10's to date, then the market could survive.

h. Is it more efficient in this model for everyone to find dates online or for everyone to find dates in the real world? Why is it not possible to implement the efficient outcome?

Online. Because of lower search costs, utility is higher with online dating, but an efficient outcome does not obtain because of the asymmetric information inherent in the online world.

i. In the used car market, a common way to ameliorate the information problem is to have a car inspected. What is the most common way on dating websites to ameliorate the information problem (i.e. so that "quality 10" person doesnt end up on a date with a "quality 1" person)?

Photos.

j. What kinds of information problems in the dating market are not solved by the technique you identified in i?

Information related to quality which is not conveyed in a photo alone.