Quiz #5

Problem 1 Suppose that Agnes' utility over wealth is $\sqrt{w}$, and that she initially has wealth of \$100.

She is offered an investment opportunity that has a 50% probability of raising her wealth to \$120 and a 50% probability of lowering her wealth to \$80.

a. What is Agnes' certainty equivalent, i.e. the minimum amount of certain wealth she would prefer to the uncertain outcome of this gamble?

Agnes' expected utility from the investment is $.5\sqrt{120} + .5\sqrt{80} = 9.95$. To get this much utility from a given amount of certain wealth, she would need \$w, where $9.95 = \sqrt{w}$, so $w = \$98.99$

b. What is Agnes' risk premium, i.e. the minimum amount she would need to be compensated before she would prefer the investment to doing nothing?

The expected value of her wealth after the investment is \$100, so the risk premium is \$100-\$98.99=\$1.01

c. What term from class describes Agnes' attitude towards risk?

Agnes is risk averse.

Problem 2 Consider the following two gambles:

Gamble A	**Gamble B**
89% chance of \$100	50% chance of \$0
10% chance of \$500	30% chance of \$500
1% chance of \$0	20% chance of a free ticket to Gamble A

a. What is the expected value of Gamble A?

Gamble A has an expected value of $.89 * \$100 + .1 * \$500 + .01 * \$0 = \$139$.

b. What is the expected value of Gamble B?

Gamble B has an expected value of $.5 * \$0 + .3 * \$500 + .2 *$ Expected value of gamble A $= \$177.8$

Suppose that Omar's utility function is $u(w) = \sqrt{w}$, where w is his winnings.

c. Calculate Omar's expected utility from Gamble A.

Omar's expected utility is $.89 * \sqrt{100} + .1 * \sqrt{500} + .1 * \sqrt{0} = 11.14$.

d. Omar has a ticket for Gamble A. What is the smallest amount of money he would accept in exchange for the ticket?

Omar's certainty equivalent for gamble A is given by $11.14 = \sqrt{CE}$, or \$124.01.

(extra credit: 10 points) Calculate Omar's expected utility from Gamble B. What is the smallest amount of money he would accept in exchange for a ticket to gamble B?

Omar's expected utility from gamble B is $.5*\sqrt{0}+.3*\sqrt{500}+.2*.89*\sqrt{100}+.2*.1*\sqrt{500}+.2*.1*\sqrt{0} = 8.935$. His certainty equivalent for gamble B is then \$79.84. Note that gamble B is much less attractive to Omar, even though it has a higher expected value, as Omar is risk averse.

Problem 3 For a certain model of used car, 10% of all owners value their cars at \$20,000, 45% at \$15,000, 30% at \$10,000, and 15% at \$5,000. Suppose a buyer values any car at 1.4 times its value to the seller. However, while each seller is aware of the value of his car, all used cars appear identical to a buyer.

a. If all four types of used cars are traded, what is the highest price a buyer would be willing to pay, given that he cannot tell which type of car he is getting ahead of time? Can there be an equilibrium in which all four types are sold?

If all four types are sold, a buyer would be willing to pay at most $1.4*(.1*\$20,000+.45*\$15,000+.3*\$10,000+.15*\$5,000) = \$17,500$. As this is not enough to induce the sellers of \$20,000 cars to stay in the market, no, there cannot be an equilibrium in which all four types of cars are traded.

b. Can there be an equilibrium in which the three lowest types of used cars are sold? What is the range of prices that will support mutually beneficial trade in this case?

If the three worst types of cars are sold, then a buyer is willing to pay at most $1.4*(.5*\$15,000+\frac{1}{3}\$10,000+\frac{1}{6}\$5,000) = \$16,333.33$. This price is high enough to keep all three remaining types of sellers active in the market. There can be an equilibrium at any price between \$15,000 and \$16,333.33.