Homework 3
due Wednesday, February 29

Problem 1 Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by $C = 200 + 2q^2$, where q is the level of output and C is total cost. (The marginal cost of production is $4q$; the fixed cost is $200.)

a. If the price of watches is $100, how many watches should you produce to maximize profit?

b. What will the profit level be?

c. What is the smallest the price can be for the firm to produce in the short-run? In the long-run?

Problem 2 A competitive firm has the following short-run cost function': $C(q) = q^3 - 8q^2 + 30q + 5$.

a. Find MC, AC, and AVC, and sketch them on a graph (hint: $MC = 3q^2 - 16q + 30$.)

b. At what range of prices will the firm supply zero output, in the short-run?

c. Identify the firm's supply curve on your graph.

d. At what price would the firm supply exactly 6 units of output?

Problem 3 A firm has production function $f(K, L) = \sqrt{KL}$. In the short run, the firm has capital $K = 400$; this cannot be changed in the near future. The cost of a unit of capital is $20, while the cost of a unit of labor is $30.

a. In the short-run, how much labor does the firm need to employ in order to produce $q = 300$ output? What is the cost of producing 300 output?

b. Repeat part a for $q = 400$, $q = 500$ and $q = 600$.

c. What is the cost of producing q output in the short-run?

d. What is the marginal cost of producing a 301st unit? A 401st unit? If you are comfortable doing so, you may answer this question by writing down the marginal cost function directly, rather than recalculating total cost for $q = 301$ and $q = 401$.

e. Given your answer to part c, draw a graph with the firm's average total cost, average variable cost, and marginal cost (hint: $MC = \frac{3}{20}q$).

f. Suppose the firm operates in a competitive market, and the price of the output good is $p = 15. How much output will the firm supply. What will the firm's profit be at this price?

g. Now suppose that the competitive price changes, to $p. How much output will the firm supply, as a function of p? What is the minimum price that the firm needs in order to be profitable in the short-run? In the long-run?

h. Finally, suppose now the firm's marginal cost is still $MC = \frac{3}{20}q$, but the firm is a monopoly with demand curve equal to $p = 24 - \frac{3}{40}q$. Solve for the firm's profit-maximizing price and quantity. What is the monopolist's profit?

Problem 4 Magee's Bakery, in downtown Lexington, estimates that its demand for transparent pies has a price elasticity of -1.5.

a. Suppose Magee's were to increase its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation.

b. Suppose Magee's were to decrease its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation.

c. Finally, suppose for part c only that you now have the additional information that the marginal cost to Magee's of making one transparent pie is $2 (and is constant). Magee's currently charges $5 for each transparent pie. Should they increase or decrease this price, or should they leave it at $5?

Problem 5 Answer the following two questions about perfectly competitive markets:

a. There are many taxi drivers in New York City, all of whom sell identical taxi rides. Taxi drivers in New York are required to hold a special license to operate, and the government only issues 5,000 licenses. Will these drivers necessarily earn zero profit in the long run, or is it possible for them to earn positive profit?

b. Carlos owns a gas station in Lexington. Carlos estimates his total costs are given by $TC = 400 + .01q^2$ and his marginal costs are given by $MC = .02q$. The price of gas is currently $3.75/gallon. Should Carlos stay in business in the long run? In the short run?

Problem 6 John runs the only carwash in town; he is a monopolist. John estimates his daily demand for carwashes is given by the expression $Q = 100 - 4P$, where Q is the number of carwashes drivers will purchase at price P. It costs John $5 in electricity, soap, etc to run the carwash once. Additionally, John has fixed costs of $300/day.

a. What price should John set for a carwash? What will be his daily profit at this price?

b. If John were to lower his price by $1, he would sell more carwashes, and still be able to charge a price above his marginal cost. Explain intuitively why it would not be profit-maximizing to do so.

c. What is John's elasticity of demand at his profit-maximizing price? Is it elastic or inelastic? If it is elastic, why does he not lower his price, as this would surely bring in many more customers? If inelastic, why does he not raise his price?

Problem 7 The Phoenix Moons, a pro football team, have a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is $30, yet this results in an average attendance of only 20,000 people.

a. Explain how it can be profitable to leave 10,000 seats empty.

b. Next week the Moons play the Tucson Turkeys, who have offered to buy an unlimited number of tickets at $4 each, to be resold only in Tucson. How many tickets should be sold to Tucson to maximize profits — 10,000? More than 10,000? Fewer than 10,000?

c. Given your answer to b., what price should the Moons charge their own fans? $4? $5? More than $5?