Economics 590, fall 2007

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Homework 4
due 10/2/2007

Problem 1 (Weierstrass theorem). *(Sundaram, page 98 #13) A monopolist faces a downward sloping inverse-demand curve $p(x)$ that satisfies $p(0) < \infty$ and $p(x) \geq 0$ for all $x \in \mathbb{R}_+$. The cost of producing x units is given by $c(x) \geq 0$, where $c(0) = 0$. Suppose $p(\cdot)$ and $c(\cdot)$ are both continuous on $\mathbb{R}_+$. The monopolist wishes to maximize profit, $\pi(x) = xp(x) - c(x)$, subject to the constraint $x \geq 0$.*

a) Suppose there is $x^* > 0$ such that $p(x^*) = 0$. Show that the Weierstrass theorem can be used to prove the existence of a solution to this problem.

b) Now suppose instead there is $\tilde{x} > 0$ such that $c(x) \geq xp(x)$ for all $x \geq \tilde{x}$. Show, once again, that the Weierstrass theorem can be used to prove existence of a solution.

c) What about the case where $p(x) = \bar{p}$ for all x (the demand curve is infinitely elastic) and $c(x) \to \infty$ as $x \to \infty$?

Problem 2 (Weierstrass theorem II). *(Sundaram, page 97 #2) Suppose $A \subset \mathbb{R}^n$ is a set consisting of a finite number of points $\{x_1, x_2, ..., x_p\}$. Show that any function $f : A \to \mathbb{R}$ has a maximum and a minimum on A. Is this result implied by the Weierstrass theorem? Explain.*

Problem 3 (Weierstrass theorem III). *(Sundaram, page 97 #1) Prove or counter the following statement:*

If f is a continuous real-valued function on a bounded (but not necessarily closed) set A, then $\sup f(A)$ is finite. (nb. $\sup f(A) = \{y \in \mathbb{R} : y = f(x) \text{ for some } x \in A\}$).

Problem 4 (Sequences). *(Sundaram, page 67 #3) Let $\{x_n\}$, $\{y_n\}$ be sequences in $\mathbb{R}^n$ such that $x_n \to x$ and $y_n \to y$. For each n, let $z_n = x_n + y_n$, and let $w_n = x_n * y_n$. Show that $z_n \to (x + y)$ and $w_n \to x * y$.*

Problem 5 (Sequences II). *Give an example of a sequence in Euclidean space $(\mathbb{R}, |\cdot|)$ which has*

a) exactly zero subsequential limits

b) exactly one subsequential limit

c) exactly two distinct subsequential limits

d) exactly three distinct subsequential limits

e) exactly n distinct subsequential limits, for $n \in \mathbb{Z}$.

(note: for problem 5, you do not need to show any work)

Problem 6 (Sequences III). *For this question, you may use the fact that if $\{x_n\}$ is increasing ($x_{n+1} > x_n$) and is bounded above, then $x_n \to \sup\{x_n\}$, while if $\{x_n\}$ is decreasing and bounded below, $x_n \to \inf\{x_n\}$.*

Prove that the following sequences converge:

a) $\{x_n : x_n = \frac{n^2}{n^2+1}\}$

b) $\{x_n : x_n = \frac{n^2-n}{n^3+1}\}$

c) $\{x_n : x_n = \frac{\sin(n)}{n}\}$

d) $\{x_n : x_n = \frac{2^n}{n!}\}$