$$f(k,\ell) = 4k^{\frac{3}{4}}\ell^{\frac{1}{4}}$$

$$\nabla f(k,\ell) = \begin{bmatrix} 3k^{-\frac{1}{4}}\ell^{\frac{1}{4}} \\ k^{\frac{3}{4}}\ell^{-\frac{3}{4}} \end{bmatrix} \qquad D^2 f(k,\ell) = H(k,\ell) = \begin{pmatrix} -\frac{3}{4}k^{-\frac{5}{4}}\ell^{\frac{1}{4}} & \frac{3}{4}\ell^{-\frac{3}{4}}k^{-\frac{1}{4}} \\ \frac{3}{4}k^{-\frac{1}{4}}\ell^{-\frac{3}{4}} & -\frac{3}{4}k^{\frac{3}{4}}\ell^{-\frac{7}{4}} \end{pmatrix}$$

$$x'H(k,\ell)x = -\frac{3}{4}k^{-\frac{5}{4}}\ell^{\frac{1}{4}}x_1^2 + \frac{3}{4}k^{-\frac{1}{4}}\ell^{-\frac{3}{4}}x_1x_2 + \frac{3}{4}\ell^{-\frac{3}{4}}k^{-\frac{1}{4}}x_1x_2 - \frac{3}{4}k^{\frac{3}{4}}\ell^{-\frac{7}{4}}x_2^2$$

$$; \; x'H(k,\ell)x \le 0 \iff 2k^{-\frac{1}{4}}\ell^{-\frac{3}{4}}x_1x_2 \le k^{-\frac{5}{4}}\ell^{\frac{1}{4}}x_1^2 + k^{\frac{3}{4}}\ell^{-\frac{7}{4}}x_2^2$$

$$\iff 2k\ell x_1x_2 \le \ell^2x_1^2 + k^2x_2^2 \quad \left(\text{multiply both sides by } k^{\frac{5}{4}}\ell^{\frac{7}{4}}\right)$$

$$\iff 0 \le (\ell x_1 - kx_2)^2,$$ which is clearly true,

so $D^2f(k,\ell)$ is negative semidefinite,

and thus $f(k,\ell)$ is concave.