Homework 6

due 10/27/08

Problem 1 (Unconstrained optimization) (Sundaram, page 110)

Find and classify the critical points (local minimum, local maximum, neither) of each of the following functions on $\mathbb{R}^2$. Are any of the local optima also global optima?

a. $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

b. $f(x,y) = e^{2x}(x + y^2 + 2y)$

c. $f(x,y) = xy(7 - x - y)$

d. $f(x,y) = x \sin y$

e. $f(x,y) = x^4 + x^2y^2 - y$

f. $f(x,y) = x^4 + y^4 - x^3$

g. $f(x,y) = \dfrac{x}{1 + x^2 + y^2}$

h. $f(x,y) = \dfrac{x^4}{32} + x^2y^2 - x - y^2$

Problem 2 (Unconstrained optimization II) Dingbat Airlines has regular flights between Ypsilanti and Kalamazoo. It can treat business and leisure travelers as separate markets by demanding advance purchase and Saturday night stay-over for leisure travelers. Suppose that it notes a demand function of $Q_b = 16 - p_b$ for business travelers and a demand function $Q_l = 10 - p_l$ for leisure travelers, and that it has a cost function for all travelers of $c(Q) = 10 + Q^2$, where $Q = Q_b + Q_l$. How much should it charge in each market to maximize its profit?

Problem 3 (Constrained optimization) (Sundaram, page 142)

Find the maximum and minimum of $f(x,y) = x^2 - y^2$ on the unit circle $x^2 + y^2 = 1$ using the Lagrange multipliers method. Using the substitution $y^2 = 1 - x^2$, solve the same problem as a single variable unconstrained problem. Do you get the same results? Why or why not?

Problem 4 (Constrained optimization II)

Solve each of the following constrained maximization problems. If no maximum exists on the constraint set, prove it.

a. $\max_{x \geq 0, y \geq 0} 3\log(x) + 4\log(y)$ subject to $24 - 2x - 3y = 0$

b. $\max_{x \geq 0, y \geq 0} x^{\frac{1}{2}} + y^{\frac{1}{2}}$ subject to $100 - 2x - y = 0$

c. $\max_{x \geq 0, y \geq 0} x^3 + y^3 + 3xy^2 + 3x^2y$ subject to $20 - x - y = 0$

d. $\max_{x \geq 0, y \geq 0} x + \log(y)$ subject to $p_x * x + p_y * y = m$

e. $\max_{x \in \mathbb{R}, y \in \mathbb{R}} \dfrac{1}{3}x^3 - \dfrac{3}{2}y^2 + 2x$ subject to $x - y = 0$

For part d., note that p_x, p_y and m are unknown parameters. Be sure to point out how different values will affect your answer. Also, "log" denotes the natural logarithm, sometimes written as ln.