Economics 590, fall 2007
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Final
12/11/2007

Directions: **Answer all of the questions.** This exam spans the front and back of this sheet. Carefully state the appropriate definitions and theorems, where applicable, and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the immediately preceding one. Good luck.

Problem 1 (20 points). *Consider the problem of maximizing* $f(x,y) = -(x-8)^2 - (y-4)^2$ *subject to the constraints* $10x + 5y \leq 200$, $x \geq 0$, *and* $y \geq 0$.

a. Argue that a maximum exists and will necessarily appear as one of the points satisfying the Kuhn-Tucker conditions.

b. Write down the Kuhn-Tucker conditions, and find all point(s) satisfying the Kuhn-Tucker conditions. Make sure to solve for the values of any multipliers as well.

c. Which of the point(s) you solved for in b. maximizes f *on the given constraint set?*

Problem 2 (15 points). *Let* $f : [0,10] \to \mathbb{R}$ *be a convex function. Suppose that* f *obtains a maximum at* $x^* \in (0,10)$. *Prove that* $f(x) = f(x^*)$ *for all* $x \in [0,10]$.

Problem 3 (20 points). *Consider the problem of maximizing* $\sum_{t=0}^{\infty} \left(\frac{2}{3}\right)^t log(c_t)$ *subject to the constraint* $c_t + w_{t+1} = 20w_t^{\frac{3}{4}}$, $t = 0, 1, ...$, w_0 *given.*

a. Write down the recursive form of the problem using value functions.

b. Give a closed-form expression for the optimal value of one period-ahead wealth, w_{t+1}.

c. Solve for the steady-state level of wealth, w^*. *Show your work.*

d. Go as far as you can in proving that any initial level of wealth $w_0 > 0$ *will eventually converge to the steady state level you solved for in part c.*

Problem 4 (20 points). *Consider the problem of choosing consumption and wealth sequences to maximize*

$$\sum_{t=0}^{\infty} \beta^t \left(log(c_t) + \gamma \log(c_{t-1}) \right) \ \textit{subject to } c_t + w_{t+1} = A w_t^{\alpha}$$

where $\beta \in (0,1)$, $\gamma > 0$, $\alpha \in (0,1)$, *and* w_0 *and* c_{-1} *are given.*

Here, c_t *denotes consumption at time* t, w_t *is wealth saved for time* t. *Note that consumption purchases generate a stream of utility that extends for two periods.*

a. Write the value function $V[w, c_{-1}]$ *associated with the maximization problem.*

b. Guess that the value function has the form $V[w, c_{-1}] = a + b\log(w) + d\log(c_{-1})$; *verify your guess and solve for the coefficients* a, b, d.

c. Give expressions for optimal choices of consumption c *and one-period-ahead wealth* w', *as functions of state variables* w *and* c_{-1}.

Problem 5 (15 points). *Solve the following maximization problem:*

$$\max_{x,y,z} x + y + z \ \textit{subject to } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Problem 6 (10 points). *Solve the following maximization problem:*

$$\max_{x,y} x^2 + y^2 - 2x + 1 \ \textit{subject to } x^2 + 4y^2 = 16$$