Homework 5
due 11/15/11, in class

Problem 1 (hyperbolic discounting) Economists say that "hyperbolic discounting" describes the behavior of someone saying 'today I'll spend, and tomorrow I'll save' or 'I'll eat a big meal today, and start dieting tomorrow', day after day. Such attitudes are not time consistent, because when tomorrow rolls around, the person says the same thing. This type of model has been used to explain the behavior of smokers and the low US savings rate (see the papers of David Laibson).

Consider the following model of hyperbolic discounting, where $\gamma \in (0,1)$ and $\beta \in (0,1)$ are both discount factors.

$$\max_{\{c_t, w_{t+1}\}_{t=0}^{\infty}} log(c_0) + \gamma \left[\beta * log(c_1) + \beta^2 * log(c_2) + \beta^3 * log(c_3) + ... \right]$$

$$\text{subject to } c_t + w_{t+1} = Aw_t^{\alpha} \text{ for } t = 0, 1, 2, ...$$

$$w_0 \text{ given}$$

note that this is identical to the model solved in the 10/18 class except for the unusual discounting.

a. Go as far as you can in explaining why the above model does or does not fit the 'today I'll spend, but tomorrow I'll save' story discussed above.

b. Write down a Bellman equation $V[w]$ corresponding to the above problem (Hint: look at the example we did in the 10/18 class and note that the period t maximization problem is not the same as the period $t+1$ problem from the point of view of period t, so you need two different value functions).

c. Derive a closed form solution for $V[w]$, using the guess-and-verify approach (again, look carefully at the standard model studied in the 10/18 class).

d. Define the *savings rate* to be $\frac{w'}{Aw^{\alpha}}$ to be the fraction of all output saved for the subsequent period. Does this savings rate change over time? Show that the actual savings rate in period t is different from that which was planned from the perspective of period $t-1$.

Problem 2 (Guess and verify) A vintner[1] has one unit of labor to use each day. He can allocate that labor between the making of bread and the pressing of grapes for grape juice. The bread he makes today he can consume today. The grape juice he makes today will become tomorrow's wine (he doesn't care for grape juice). The production technology is linear: it produces one unit of bread per unit of labor allocated to baking, and one unit of juice per unit of labor allocated to grape pressing, and one unit of wine per unit of grape juice left to ferment. The transformation of juice into wine requires no labor, only time. The vintner allocates his labor so as to maximize the utility of his own consumption. His utility function has the form:

$$\sum_{t=0}^{\infty} \beta^t \sqrt{b_t w_t}$$

where b_t and w_t are the bread and wine consumption, respectively, in period t. The initial wine consumption w_0 is given. The discount factor is $\beta \in (0,1)$.

a. Write down the Bellman equation associated with this problem. What are the control variable(s)? State variable(s)?

---

[1]vintner=one who makes wine

   b. Guess that the value function has the form $V(w) = \alpha\sqrt{\gamma + w}$, where α and γ are unknown parameters, and verify that V has this form. Solve for parameters α and γ.

   c. State the optimal choice of next period's wine, w', as a function of current wine w.