Midterm Exam

answers

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 90 minutes to complete the exam. Good luck!

Problem 1 Consider the following game:

	Y	Z
W	a,b	c,d
X	e,f	g,h

a. List all inequalities that must hold for (W, Y) to be a dominant strategy equilibrium.

$a > e$, $c > g$, $b > d$, $f > h$

b. List all inequalities that must hold for (W, Y) to be a Nash equilibrium.

$a \geq e$, $b \geq d$

Problem 2 Consider the following game:

		Stringer	
		L	R
Avon	T	4,8	0,0
	B	8,20	X,Y

a. If (B, R) is the Nash equilibrium of this game, what must be true of X and Y? Your answer should be two inequalities, one for X and one for Y.

$X \geq 0$, $Y \geq 20$

b. If this game is played sequentially, with Avon moving first and (B, R) is the subgame perfect equilibrium outcome, what must be true of X and Y? Your answer should be two inequalities, one for X and one for Y.

$X \geq 4$, $Y \geq 20$

Problem 3 This problem refers to the following game:

	A	B	C
A	4,4	0,5	-1,0
B	5,0	1,1	0,0
C	0,-1	0,0	1,1

a. What are the pure-strategy Nash equilibria?

(B, B), and (C, C).

b. Is there a mixed strategy Nash equilibrium where both players mix A and B? If so, find the equilibrium. If not, explain why not.

No. B strictly dominates A for player 1, yet in a mixed Nash equilibrium 1 would need to be indifferent between A and B, which is impossible.

c. Is there a mixed-strategy Nash equilibrium where both players mix B and C? If so, find the equilibrium. If not, explain why not.

There is a mixed strategy Nash equilibrium, in which both players play $(\frac{1}{2}B + \frac{1}{2}C)$.

Problem 4 Consider an infinitely repeated version of the game below:

		Player 2	
		C	D
Player 1	C	7,7	4,8
	D	8,4	5,5

Suppose both players use the following strategy:

(I) Play C initially, and as long as (C, C) has been played in every previous period.

(II) If any player has ever deviated from (I), play D forever after.

a. Suppose both players share a common discount factor, δ. For what values of δ do the above strategies comprise a subgame perfect equilibrium of the repeated game?

Since the game is symmetric, it suffices to check the incentive constraint of only player 1. If player 1 does not deviate in phase I, she gets a payoff of $\frac{7}{1-\delta}$, while if she does, she gets $8 + \delta\frac{5}{1-\delta}$. Therefore, she prefers not to deviate if $\delta \geq \frac{1}{3}$. Since the punishment path consists of repeated play of a stage game Nash equilibrium, it is trivial that neither player wishes to deviate once in phase II. Conclude that the proposed strategies comprise a SPE if and only if $\delta \geq \frac{1}{3}$.

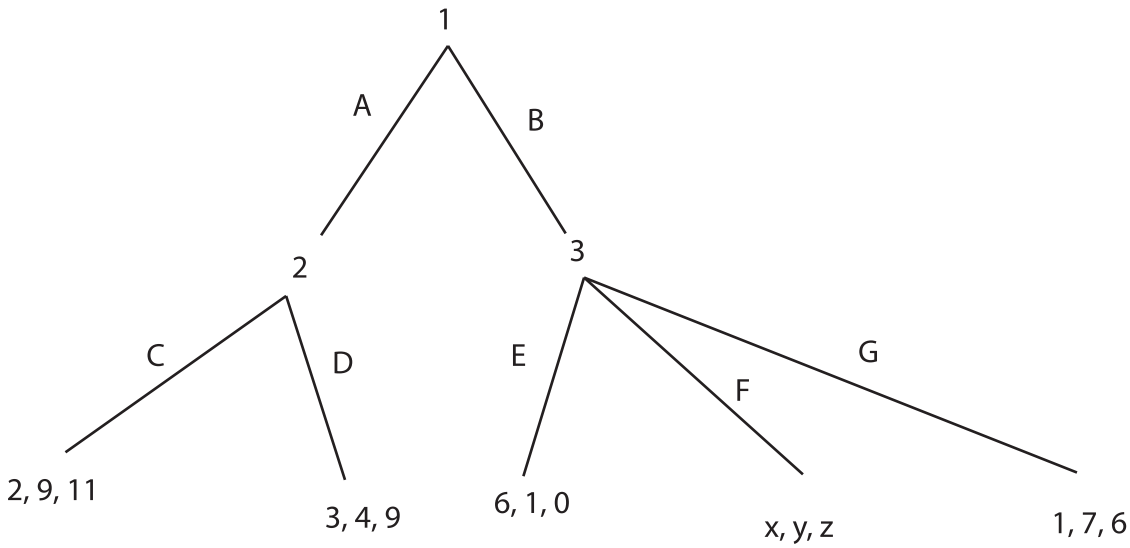
b. Suppose that the payoffs for (D, D) increased to $(6, 6)$. In this new game, for what values of δ do the above strategies comprise a subgame perfect equilibrium of the repeated game?

By the same logic in part a, the proposed strategies comprise a SPE if and only if $\delta \geq \frac{1}{2}$.

c. Is it easier to sustain cooperation in part d. or part e.? Provide some intuition for your answer.

It is easier to sustain cooperation in part a, in the sense that players need not be as patient as in part b. The reason is that the punishment path payoffs are higher in part b, meaning that players suffer less in phase II of the game. But this lowers the cost to deviating, which means relatively impatient players will be more tempted to deviate in phase I to get a short-term increase in utility.

Problem 5 Consider the game below.



a. What must be true of x, y, and z for outcome F to occur in a subgame perfect equilibrium of this game?
   $z \geq 6$, $x \geq 2$

b. Suppose $z = 5$ and $x = y = 20$. What is the subgame perfect equilibrium of this game?
   1 plays A, 2 plays C, 3 plays G

Problem 6 Consider the infinitely repeated version of the following stage game:

		Player 2 A	Player 2 B
Player 1	A	5,5	-2,10
	B	10,-2	0,0

Suppose each player plays the following strategy:

(Phase I) Play A initially; remain in phase I so long as no deviation occurred in the previous period. If either player deviates, move to Phase II in the following period.

(Phase II) Play (B, B) for T periods. If either player deviates, restart Phase II. After T periods, return to Phase I.

Suppose that both players have a common discount factor, $\delta = \frac{4}{5}$. Find the minimum value of T for which the above strategies comprise a subgame perfect equilibrium of the repeated game. Most of the points will be awarded for correctly describing the inequality which determines this δ.

Since (B, B) is a stage game Nash equilibrium, it is trivial that neither player will wish to deviate during the T-period punishment phase. To check whether either player wishes to deviate in phase I, it suffices to check player 1's incentive constraint, since the game is symmetric. If player 1 sticks to the prescribed strategy, she gets $\frac{5}{1-\delta}$, while if she deviates she gets $10 + \delta^{T+1}\frac{5}{1-\delta}$. The former is greater than the latter so long as $T \geq 2$.