Final Exam

5/4/2015

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. The six questions are equally weighted.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 150 minutes to complete the exam. Good luck!

Problem 1 Suppose that *qualified* and *unqualified* workers look alike, but perform differently on a pre-employment test. Specifically, suppose that an unqualified worker's test score is drawn from a $U[0, \frac{3}{4}]$ distribution, while a qualified worker's score is drawn from a $U[\frac{1}{2}, 1]$ distribution.

a. Suppose that prior to seeing a worker's test score, a firm believes a worker is qualified with probability π. What is the firm's posterior belief that a worker is qualified if his score is between 0 and $\frac{1}{2}$? Between $\frac{1}{2}$ and $\frac{3}{4}$? Between $\frac{3}{4}$ and 1?

Between 0 and $\frac{1}{2}$: 0. Between $\frac{1}{2}$ and $\frac{3}{4}$: $\frac{\frac{1}{2}\pi}{\frac{1}{2}\pi+\frac{1}{3}(1-\pi)}$. Between $\frac{3}{4}$ and 1: 1.

b. Suppose that a firm can place a worker into a good job or a bad job. If a qualified worker is placed into a good job, the firm earns a profit of $\$4,000$, while if an unqualified worker is placed into a good job, the firm loses $\$9,000$. If a worker is placed into a bad job, the firm breaks even.

Consider a worker whose test score is between $\frac{1}{2}$ and $\frac{3}{4}$. Characterize the firm's decision to place the worker into a good or a bad job as a function of π.

Good job iff $\pi \geq \frac{3}{5}$.

c. Suppose the firm places a worker into a good job if and only if the worker scores at least $\frac{1}{2}$ on the test. Suppose further that the cost c of becoming qualified varies across workers, following $c \sim U[0, 1]$, and that the net benefit to a worker of being in a good job is $\omega = 1$. What fraction of workers will become qualified?

$\frac{2}{3}$

d. Now suppose the firm places a worker into a good job if and only if the worker scores at least $\frac{3}{4}$ on the test. What fraction of workers become qualified?

$\frac{1}{2}$

e. Determine whether one or both of the scenarios of parts c-d constitute an equilibrium.

Both.

Problem 2 Workers choose their education levels and then apply for jobs. $\frac{1}{4}$ of workers are *high-ability*, while the remainder are *low-ability*. Firms cannot determine a given worker's ability level until after the worker is hired. Regardless of education level, high ability workers increase firm profits by \$100, while low ability workers increase firm profits by \$36. The cost of education is $c_L(e) = e^2$ for low-ability workers, while it is $c_H(e) = 2e$ for high ability workers. Assume that workers are paid their expected productivity.

For outcomes 1-4 below, determine if each outcome is an equilibrium of the game. If so, determine whether or not it satisfies the intuitive criterion, and prove your answer.

a. Outcome 1:

$$e_H = 12$$
$$e_L = 2$$
$$w(e) = \begin{cases} \$100 & \text{if } e = 12 \\ \$36 & \text{if } e \neq 12 \end{cases} \tag{1}$$

Not an equilibrium (low types prefer to switch to $e = 0$).

b. Outcome 2:

$$e_H = 1$$
$$e_L = 1$$
$$w(e) = \begin{cases} \$52 & \text{if } e < 25 \\ \$100 & \text{if } e \geq 25 \end{cases} \tag{2}$$

Not an equilibrium (low types prefer to switch to $e = 0$).

c. Outcome 3:

$$e_H = 5$$
$$e_L = 5$$
$$w(e) = \begin{cases} \$36 & \text{if } e < 5 \\ \$52 & \text{if } e = 5 \\ \$(32 + 4e) & \text{if } e \in (5, 17) \\ \$100 & \text{if } e \geq 17 \end{cases} \tag{3}$$

Not an equilibrium. Low types prefer to switch to $e = 0$.

d. Outcome 4. For this part, your answer should depend on the variables X and Y.

$$e_H = X$$
$$e_L = Y$$
$$w(e) = \begin{cases} \$36 & \text{if } e < Y \\ \$100 & \text{if } e \geq Y \end{cases} \tag{4}$$

Equilibrium if $X = Y$ and $Y \leq 32$. Satisfies intuitive criterion if $Y = 8$.

Problem 3 An incumbent firm is either a low-cost type (θ_L) or a high-cost type (θ_H), each with equal probability. In period 1, the incumbent is a monopolist and sets one of two prices, p_L or p_H, with its profits in period 1 given by the following table:

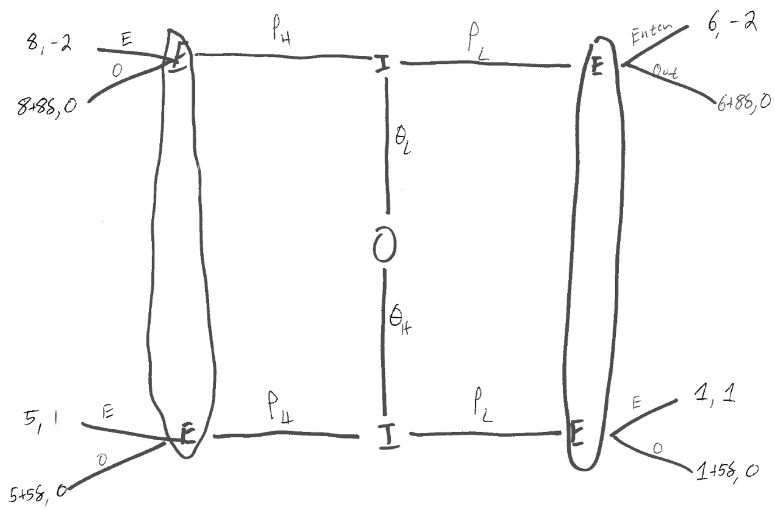
Type	Profit from p_L	Profit from p_H
θ_L	6	8
θ_H	1	5

After observing the period 1 price, a potential entrant (which does not know the incumbent's type) can choose to enter the market (E) or to stay out (O) in period 2. The payoffs of both players in period 2 are as follows:

Type	Entrant's choice	Incumbent's profit	Entrant's profit
θ_L	E	0	-2
θ_L	O	8	0
θ_H	E	0	1
θ_H	O	5	0

At the beginning of the game the incumbent discounts period 2 profits using discount factor $\delta \leq 1$.

a. Draw the extensive form game. Make sure to include all payoffs, as a function of δ.



Note that it is also acceptable to multiply the entrant's payoffs by δ. Since they all occur in period 2, however, doing so is unnecessary.

b. For the special case of $\delta = 1$, find a pooling perfect Bayesian equilibrium in which both incumbent types choose p_L in period 1. Does your equilibrium satisfy the intuitive criterion?

I plays E at his left information set and O at his right information set. Both types of I play P_L. Entrant believes he is at the bottom node of his left information set and is equally likely to be at the top and bottom nodes of his right information set.

c. Find the range of discount factors for which a separating equilibrium exists in which type θ_L chooses p_L and type θ_H chooses p_H in period 1.

$\delta \in [\frac{1}{4}, \frac{4}{5}]$.

Problem 4 Consider the game in Figure 1 below.

a. Draw the reduced normal form. Find all pure strategy Nash equilibria. There is a mixed Nash equilibrium in which 1 randomizes between A and B, and 2 randomizes between L and R. Find it.

One pure Nash equilibrium, at (C, M). The mixed Nash equilibrium is at $(\frac{1}{2}A + \frac{1}{2}B, \frac{1}{2}L + \frac{1}{2}R)$.

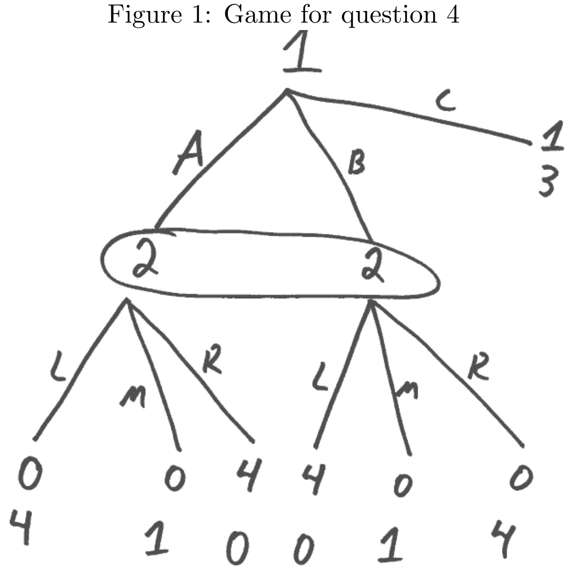
b. Find all of the game's perfect Bayesian equilibria (pure as well as mixed).

The one PBE is $(\frac{1}{2}A + \frac{1}{2}B, \frac{1}{2}L + \frac{1}{2}R)$, with 2 believing that each node is equally likely.

c. Explain in intuitive terms any differences between your answers to part a and part b.

The pure Nash equilibrium involves 2 playing a strictly dominated strategy. PBE rules this out.

Figure 1: Game for question 4



Problem 5 Consider two Cournot oligopolists, facing demand curve $P = 1 - q_1 - q_2$. Firm 1's marginal cost is as follows:

$$c_1 = 0 \text{ w.p. } (1-\alpha) \text{ (Firm 1 is low cost)}$$
$$c_1 = X \text{ w.p. } \alpha \text{ (Firm 1 is high cost)}$$

Firm 1 knows its marginal cost, but Firm 2 knows only the distribution given above. Firm 2 has marginal cost equal to 0.

a. Solve for Firm 1's best response functions. Note that since there are two types of Firm 1 (high and low cost), Firm 1 has two best response functions.

Low cost Firm 1: $q_1^L = \frac{1}{2} - \frac{1}{2}q_2$. High cost Firm 1: $q_1^H = \frac{1}{2} - \frac{1}{2}q_2 - \frac{1}{2}X$.

b. Solve for Firm 2's best response function.

$q_2 = \frac{1}{2} - (1-\alpha)\frac{1}{2}q_1^L - \alpha\frac{1}{2}q_1^H$.

c. In the oligopoly game's Bayesian Nash equilibrium, what quantity does Firm 2 produce? What quantity does Firm 1 produce if its costs are low? If its costs are high? What is the market price in each case?

$$q_1^L = \frac{1}{3} - \frac{1}{6}\alpha X$$
$$q_1^H = \frac{1}{3} - \frac{1}{6}\alpha X - \frac{1}{2}X$$
$$q_2 = \frac{1}{3} + \frac{1}{3}\alpha X$$

If Firm 1's costs are high, the price is $P^H = \frac{1}{3} - \frac{1}{6}\alpha X + \frac{1}{2}\alpha X$. if Firm 1's costs are low, the price is $P^L = \frac{1}{3} - \frac{1}{6}\alpha X$.

d. What is the derivative of q_1 with respect to X in the case that Firm 1 is a low cost firm? In the case Firm 1 is high cost? Why does Firm 1's quantity depend on X even in the former case?

$\frac{\partial q_1^L}{\partial X} = -\frac{1}{2}\alpha$. $\frac{\partial q_1^H}{\partial X} = -\frac{1}{2}\alpha - \frac{1}{2}$. Both are negative. Even when Firm 1 is low-cost, a higher value of X causes Firm 2 to produce more, which causes Firm 1 to produce less.

Problem 6 Larry owns a gold mine, which contains X gold. Larry knows X, but the rest of the world knows only that $X \sim U[0,1]$.

Larry can mine the gold himself, which would give him a payoff of $3X$. He could also sell to a mining company. Since the mining company is more efficient at extracting gold, the mining company's payoff would be $4X - P$, where P is the purchase price (assume the mining company is risk neutral). If Larry sells the mine, his payoff equals P.

a. Consider a given price P. For what values of X would Larry prefer to sell? For what values would he prefer to mine himself?

Larry would sell if and only if $P \geq 3X$, or $X \leq \frac{P}{3}$.

Now consider the following two-stage pricing game. In stage one, the mining company makes a take-it-or-leave-it offer P. In stage two, Larry accepts or rejects the offer, and payoffs are realized.

b. Solve for the Bayesian Nash equilibrium of this game. What is Larry's payoff, as a function of X? What is the mining company's expected payoff (before it knows what X is)?

The unique PBE is for the mining company not to make an offer (or, equivalently, to offer $P \leq 0$). Suppose there were an equilibrium in which the mining company offered P. Given the answer to part a, the mining company's expected payoff is then $E(4X - P|X \leq \frac{P}{3}) = -\frac{1}{3}P < 0$. hence, the mining company is better off not making an offer.

c. Suppose that a third-party mining expert can accurately determine X and report it to the mining company. The cost of this service is C. For what values of C would the mining company choose to hire the third-party expert?

If the mining company knew X, it would offer Larry $P = 3X$ (or epsilon more) and he would accept. The mining company's payoff would then be $4X - 3X = X$, and so, in expectation, learning the value of X increases the mining company's profits by $EX = \frac{1}{2}$. Thus, the mining company would pay the expert if and only if $C \leq \frac{1}{2}$.