Midterm Exam

3/24/2016

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup, an intuitive explanation, or the right approach demonstrated on a simplified version of the problem. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 90 minutes to complete the exam. Good luck!

Problem 1 (10 points) Consider the following game:

		Player 2		
		L	C	R
	U	5,1	1,4	1,0
Player 1	M	3,2	0,0	3,5
	D	4,3	4,4	0,3

a. Demonstrate that L is a strictly dominated strategy for player 2.

L is dominated by $\frac{1}{2}C + \frac{1}{2}R$ (among others). If 2 plays $\frac{1}{2}C + \frac{1}{2}R$, her payoff is 2 if 1 plays U (versus 1 if 2 plays L), is 2.5 if 1 plays M (versus 2 if 2 plays L), and is 3.5 if 1 plays D (versus 3 if 1 plays D. Since $\frac{1}{2}C + \frac{1}{2}R$ yields a higher payoff than L against any pure strategy of player 1's, L is a strictly dominated strategy.

b. Demonstrate that, after removing L, U is a strictly dominated strategy for player 1.

For player 1, $\frac{1}{2}M + \frac{1}{2}D$ gives payoffs of 2 and 4, respectively, against 2's strategies of C and R, whereas U always gives player 1 a payoff of 1. Hence, U is a strictly dominated strategy after the elimination of strategy L.

c. Solve for all Nash equilibria of this game, mixed as well as pure.

By parts a. and b., there are no Nash equilibria in which player 1 plays U or player 2 plays L. There are three Nash equilibria: (D, C), (M, R), and $(\frac{1}{6}M + \frac{5}{6}D, \frac{3}{7}C + \frac{4}{7}R)$.

Problem 2 (10 points) Consider the infinitely-repeated version of the following game:

		Player 2		
		L	C	R
	T	6,6	-1,7	-2,8
Player 1	M	7,-1	4,4	-1,5
	B	8,-2	5,-1	0,0

a. For which values of discount factor δ can the players support the pair of actions (M, C) played in every period? Your answer should fully describe the strategy used.

δ must be at least $\frac{1}{5}$.

b. For which values of discount factor δ can the players support the pair of actions (T, L) played in every period (again, fully describe the strategy used)? Why is your answer different than for part a.?

δ must be at least $\frac{1}{4}$. The answer is different because there is a greater incentive to deviate from (T, L) (a gain of 2 for each player, versus only a gain of 1 for deviating from (M, C).

Problem 3 (20 points) 99 shepherds share a common field in which they graze their sheep. Each shepherd purchases as many sheep as he/she likes, at a cost of $c =$ \$300/sheep. The value of one sheep is given by:

$$v(G) = 2000 - S$$

where S is the total number of sheep which graze in the field (more sheep mean less grass/sheep, more sheep fights, etc). The common field is the only suitable location for grazing, and sheep die without grazing, so you may assume that all purchased sheep are brought to graze in the field.

a. In a symmetric Nash equilibrium, how many sheep does each shepherd purchase? How much profit is earned by each shepherd?

Each shepherd will purchase 17 sheep, and earn a profit of \$289.

b. What is the socially optimal number of sheep? If the resulting total profit is split evenly amongst all shepherds, what is the profit for each shepherd?

In a social optimum, there are 850 total sheep, or 8.58/shepherd. If the total profit from 850 sheep is split among the 99 shepherds, each will earn \$7,297.98.

c. Suppose a government imposes a tax on sheep of \$T/head, but that the revenue collected from the tax is distributed evenly to each of the 99 shepherds, regardless of how many sheep the shepherd owns. Is such a tax always welfare-reducing? Why or why not?

Such a tax can internalize the externality created when a shepherd purchases an additional sheep (namely, that the sheep lower the value of the other 98 shepherds' sheep). Specifically, a tax of T will change the equilibrium number of sheep purchased by each shepherd to $17 - \frac{T}{100}$. So long as $T \leq \$841.41$ (the tax which would produce the socially optimal number of sheep in Nash equilibrium), the tax is efficient. Even if $T > \$841.41$, the tax may be preferable to no tax.

Problem 4 (20 points) Consider a market with inverse market demand given by $P = 10 - \frac{1}{100}Q$. Firm A is a monopoly producer, with marginal cost equal to \$2.

a. Calculate Firm A's optimal quantity, and its profit as a monopolist.

Firm A produces $Q = 400$, so that the market price is \$6, and Firm A's profit is \$1,600.

Now, suppose that Firm A has discovered a new technology that will allow it to produce at a marginal cost of \$0. Implementing the new technology will cost firm A to incur a fixed cost of \$1,000.

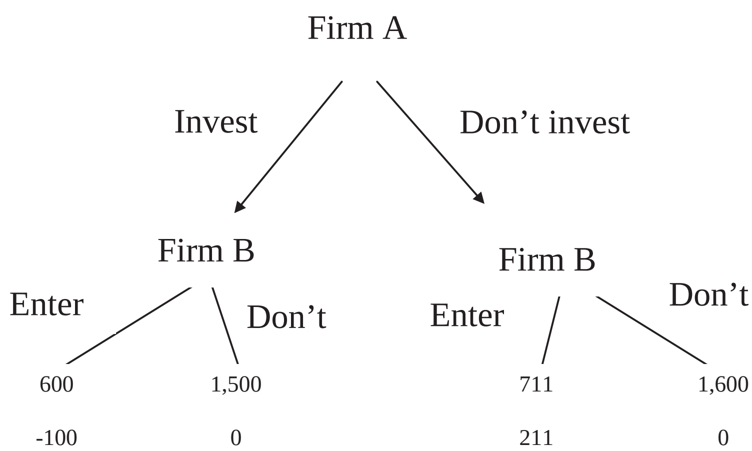
b. Is it profitable for Firm A to implement the new technology?

If Firm A has zero marginal cost, it will produce $Q = 500$, the market price is \$5, and Firm A's variable profit is $\$2,500$. Since this is only \$900 more than its profit in part a. with a higher marginal cost, Firm A will choose not to invest in the new technology.

Now, suppose that Firm A learns that Firm B is considering entering the market to compete with Firm A. To enter, Firm B would have to construct a factory at a cost of \$500, and then Firm A and Firm B would compete in Cournot oligopoly.[1] If firm B entered, its marginal cost would also equal \$2.

[1] So that inverse market demand is given by $P = 10 - \frac{1}{100}(q_1 + q_2)$, where q_i is firm i's quantity.

Figure 1: Relevant figure for question 4.d.



c. Calculate the market price, firm A's profit, and firm B's profit under Cournot competition. Would firm B profitably enter the market? (Assume for part c. that Firm A has not implemented the new technology.)

In Cournot equilibrium, Firms A and B each produce quantity $\frac{800}{3}$, the market price is \$4.67, and each firm earns a profit of \$711.11.

d. Consider an extensive form game with two rounds. In round 1, Firm A decides whether or not to implement the new technology. In round 2, Firm B decides whether or not to enter the market. Using your answers above (and possibly new calculations), determine the subgame perfect equilibrium of this game.

See Figure 1 at the end of this answer key. Note that the payoffs in the event Firm A invests and Firm B enter require an additional calculation. The SPE of this game is for Firm A to invest in the new technology, and for Firm B to not enter the market.

e. Policymakers sometimes worry that monopolists are less likely to innovate than firms in a competitive market.[2] Do your answers above suggest any caveats to this view?

Potential entrants may drive a monopolist to innovate to discourage entry.

Problem 5 (20 points) Consider the infinitely repeated version of the following stage game:

		Player 2 A	Player 2 B
Player 1	A	2,2	-1,3
Player 1	B	3,-1	1,1

Suppose each player plays the following strategy:

[2]See e.g. "Enhanced market power can also be manifested in... diminished innovation.", page 2 of *Horizonal Merger Guidelines*, 2010, U.S. Department of Justice and Federal Trade Commission.

(Phase I) Play A initially; remain in phase I so long as no deviation occurred in the previous period. If either player deviates, move to Phase II in the following period.

(Phase II) Play (B, B) for T periods. If either player deviates, restart Phase II. After T periods, return to Phase I.

Suppose that both players have a common discount factor, $\delta = .6$. Find the minimum value of T for which the above strategies comprise a subgame perfect equilibrium of the repeated game. Most of the points will be awarded for correctly describing the inequality which determines this δ.

I will give a full analytical solution. However, note that it works just as well to write down the first line of the incentive constraint and then simply try $T = 1, T = 2$, etc., until the incentive constraint holds, and this is the solution method I expect most students will use (and get full points for).

Neither player wishes to deviate so long as:

$$\begin{aligned} \frac{2}{1-\delta} &\geq 3 + \delta \sum_{t=0}^{T-1} \delta^t + \frac{\delta^{T+1}}{1-\delta} * 2 \\ \iff 2\left(\frac{1-\delta^{T+1}}{1-\delta}\right) &\geq 3 + \delta \frac{1-\delta^T}{1-\delta} \\ \iff 2 - 2\delta^{T+1} &\geq 3 - 3\delta + \delta - \delta^{T+1} \\ \iff \delta(2-\delta^T) &\geq 1 \end{aligned} \tag{1}$$

Plugging in the given value $\delta = .6$, final becomes:

$$\begin{aligned} & 1.2 - .6^{T+1} \geq 1 \\ \iff & .2 \geq .6^{T+1} \\ \iff & ln(.2) \geq (T+1)ln(.6) \\ \iff & T \geq \frac{ln(.2)}{ln(.6)} - 1 \\ \iff & T \geq 2.15 \end{aligned}$$

Since T must be an integer, neither player wishes to deviate from the proposed strategy so long as $T \geq 3$.

Problem 6 (20 points) Two bidders are bidding on a bottle of Scotch in a first-price, sealed bid auction. Bidder 1 values the bottle at v_1, and bidder 2 values the bottle at v_2. Neither bidder knows the other's valuation, but each knows that $v_i \sim U[0, 1]$, and that v_1 and v_2 are independent (note that this setting is identical to the first example studied in class). Bidders simultaneously submit hidden bids; the highest bidder gets the bottle for the price he paid.

a. Show that there is a Nash equilibrium in bidding strategies in which player i bids $b_i = \frac{v_i}{2}$.

Suppose that player 2 bids $b_2 = \frac{v_2}{2}$. We will show that player 1's best response is to bid $b_1 = \frac{v_1}{2}$. By symmetry, then, player 2's best response must also be $b_2 = \frac{v_2}{2}$, and so we will have shown that the proposed bidding strategies comprise a Nash equilibrium.

Player 1 chooses b_1 to maximize his utility:

$$\max_{b_1} (v_1 - b_1) * P(\text{player 1 wins auction}) \tag{2}$$

Given that $b_2 = \frac{v_2}{2}$, and that $v_2 \sim U[0,1]$,

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > \frac{v_2}{2}) \\ &= P(2b_1 > U[0,1]) \\ &= 2b_1 \text{ (so long as } b_1 \leq \tfrac{1}{2}\text{. The probability is 1 for } b_1 > \tfrac{1}{2}.) \end{aligned}$$

Hence, initial becomes:

$$\max_{b_1}(v_1 - b_1) * 2b_1$$

which has FOC $2v_1 = 4b_1$, or $b_1 = \frac{v_1}{2}$, as was to be shown.

b. Suppose that bidder 2 is irrational, and will bid $b_2 = v_2$. Demonstrate that $b_1 = \frac{v_1}{2}$ remains the best response for player 1.

Going back to initial, we have that:

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > v_2) \\ &= P(b_1 > U[0,1]) \\ &= b_1 \end{aligned}$$

Hence, initial becomes:

$$\max_{b_1}(v_1 - b_1) * b_1$$

which has FOC $b_1 = \frac{v_1}{2}$, as was to be shown.

c. Suppose a third bidder arrives to bid on the bottle of Scotch. Like bidders 1 and 2, bidder 3's valuation is private information, but is distributed $v_3 \sim U[0,1]$. v_1, v_2, and v_3 are independent. Show that there is a Nash equilibrium in which player i bids $b_i = \frac{2v_1}{3}$.

To answer part c., you will need to use the fact that if X_1 and X_2 are independent $U[0,1]$ random variables, $P(\max\{X_1, X_2\} < x) = x^2$ for $x \in [0,1]$.

Going back to initial, we have that:

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > max\{v_2, v_3\}) \\ &= b_1^2 \end{aligned}$$

Hence, player 1's FOC is $3b_1^2 - 2b_1v_1 = 0$, or $b_1 = \frac{2}{3}v_1$, as was to be shown.