Homework 1

due 1/26/17

Problem 1 Kirt and Lila are engaged in a joint project. If person $i \in \{K, L\}$ invests effort $x_i \in [0,1]$ in the project, at cost $c(x_i)$, the outcome of the project is worth $f(x_K, x_L)$. The worth of the project is split equally by Kirt and Lila, regardless of their effort levels, so that each gets a payoff of $\frac{1}{2}f(x_K, x_L) - c(x_i)$. Suppose effort levels are chosen simultaneously.

a. Suppose $f(x_K, x_L) = 3x_K x_L$ and that $c(x_i) = x_i^2$. Find the Nash equilibrium effort levels of this simultaneous move game.

b. Is there a pair of effort levels that yield higher payoffs for both players than do the Nash equilibrium effort levels in part a.?

Problem 2 Consider the normal form game below:

		Avon: I	Avon: N
Joe	I	r, r	$r-1, 0$
	N	$0, r-1$	$0, 0$

In this game, strategy I represents investing, and strategy N represents not investing. Investing yields a payoff of r or $r-1$, according to whether the player's opponent invests or not. Not investing yields a certain payoff of 0.

Describe the set of Nash equilibria (pure and mixed) of the game for each $r \in [-2, 3]$.

Problem 3 Gibbons, problem 1.13

Problem 4 This problem demonstrates a seeming peculiarity about mixed strategy Nash equilibria. Consider the following game between the Chicago Bears' offense and the Detroit Lions' defense. Payoffs are the number of yards advanced (positive yards for Chicago are negative yards for Detroit).

		Detroit: run defense	Detroit: pass defense
Chicago	run	-2,2	5,-5
	pass	15,-15	1,1

a. Find all pure strategy Nash equilibria, if any. Then find the mixed-strategy Nash equilibrium of the game.

b. Now suppose that the Bears improve their run game by bringing Mike Ditka[1] out of retirement:

[1] While Ditka played tight end, the combination of his blocking and the downfield threat he poses as a receiver, even at 77, would help their running game immeasurably.

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	10,-10
	pass	15,-15	1,1

Find the mixed-strategy Nash equilibrium of the new game.

c. When running the football becomes a more attractive option for the Bears, do they run more often, or pass more often? Can you explain why?

Problem 5 This problem refers to the following game:

	A	B	C
A	4,4	0,5	-1,0
B	5,0	1,1	0,0
C	0,-1	0,0	1,1

a. What are the pure-strategy Nash equilibria?

b. Is there a mixed strategy Nash equilibrium where both players mix A and B? If so, find the equilibrium. If not, explain why not.

c. Is there a mixed-strategy Nash equilibrium where both players mix B and C? If so, find the equilibrium. If not, explain why not.

Problem 6 In the game below, which strategies survive iterated removal of strictly dominated strategies? What are the pure strategy Nash equilibria?

	L	C	R
T	1,3	5,4	4,2
M	2,3	3,1	3,2
B	3,5	4,7	1,4

Problem 7 99 shepherds share a common field in which they graze their sheep. Each shepherd purchases as many sheep as he/she likes, at a cost of $c = \$300$/sheep. The value of one sheep is given by:

$$v(G) = 2000 - S$$

where S is the total number of sheep which graze in the field (more sheep mean less grass/sheep, more sheep fights, etc). The common field is the only suitable location for grazing, and sheep die without grazing, so you may assume that all purchased sheep are brought to graze in the field.

a. In a symmetric Nash equilibrium, how many sheep does each shepherd purchase? How much profit is earned by each shepherd?

b. What is the socially optimal number of sheep? If the resulting total profit is split evenly amongst all shepherds, what is the profit for each shepherd?

c. Suppose a government imposes a tax on sheep of \$T/head, but that the revenue collected from the tax is distributed evenly to each of the 99 shepherds, regardless of how many sheep the shepherd owns. Is such a tax always welfare-reducing? Why or why not?