Homework 5

due 4/6/2017

Problem 1 Suppose that *low-ability* workers have productivity of D, while *high-ability* workers have productivity of A, where $A > D$. Firms cannot tell low-ability workers from high-ability workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are low-ability, and half are high-ability.

Any worker can acquire as much education as she wishes, but getting e units of education costs a low-ability worker $B * e$, where $B > 1$, and costs a high-ability worker e. Assume the labor market is competitive, so that a worker earns her expected productivity.

a. Suppose $A = 15$, $B = 4$, and $D = 1$. Does there exist a pooling equilibrium in which both high- and low-ability workers get 1 unit of education? If so, draw or describe a wage function that supports this equilibrium outcome, and determine whether or not the equilibrium satisfies the intuitive criterion. If not, explain why not.

b. Suppose $A = 15$, $B = 4$, and $D = 1$. Does there exist a pooling equilibrium in which both high- and low-ability workers get 3 units of education? If so, draw or describe a wage function that supports this equilibrium outcome, and determine whether or not the equilibrium satisfies the intuitive criterion. If not, explain why not.

c. Suppose $A = 15$, $B = 4$, and $D = 1$. Solve for a separating equilibrium which *does not* satisfy the intuitive criterion. Draw or describe a wage function that supports this outcome in an equilibrium. Clearly explain, using your graph and/or a verbal description, why this equilibrium fails the intuitive criterion.

d. For general A, B, and D, solve for the unique equilibrium which *does* satisfy the intuitive criterion as a function of A, B and D. How does the level of education obtained by the high types vary in D in this equilibrium? What is the intuition?

Problem 2 This problem asks you to consider an extension of the basic Spence model to one in which education is productive and the cost of education is convex.

Suppose that high types with education e have productivity $y(H, e) = 10 + 2e$, while low types have productivity $y(L, e) = 2 + e$. Firms cannot observe whether a worker is a high type or a low type, but know that half of all workers are of each type. A competitive labor market ensures each type of worker is paid her expected productivity. A high type can acquire e units of education at cost $c_H(e) = \frac{1}{10}e^2$, while education costs a low type $c_L(e) = \frac{1}{4}(e + 2)^2 - 1$.

a. Suppose $e_L = 0$ and $e_H = 12$. What wage function would support this outcome as a separating equilibrium? Draw a picture and/or describe using an equation.

b. Does the equilibrium you described in part a satisfy the intuitive criterion? Why or why not?

c. Draw the set of all points which give the high type utility of 5. What is the slope of the indifference curve you drew, as a function of e? Determine the point of tangency between the high type's indifference curve and the function $y(H, e)$ (note that the high type may get more or less than 5 utility at the point of tangency). Do the same for the low type's indifference curve and the function $y(L, e)$.

d. Suppose that both types choose their education level so that their indifference curve is tangent to their productivity function. Describe, using a picture and/or an equation, a wage function that would support this outcome as a perfect Bayesian equilibrium.

e. Does the equilibrium you described in part d satisfy the intuitive criterion? Why or why not?

Problem 3 Workers choose their education levels and then apply for jobs. $\frac{1}{4}$ of workers are *high-ability*, while the remainder are *low-ability*. Firms cannot determine a given worker's ability level until after the worker is hired. Regardless of education level, high ability workers increase firm profits by \$100, while low ability workers increase firm profits by \$36. The cost of education is $c_L(e) = e^2$ for low-ability workers, while it is $c_H(e) = 2e$ for high ability workers. Assume that workers are paid their expected productivity.

For outcomes 1-4 below, determine if each outcome is an equilibrium of the game. If so, determine whether or not it satisfies the intuitive criterion, and prove your answer.

a. Outcome 1:

$$e_H = 12$$
$$e_L = 2$$
$$w(e) = \begin{cases} \$100 & \text{if } e = 12 \\ \$36 & \text{if } e \neq 12 \end{cases} \tag{1}$$

b. Outcome 2:

$$e_H = 1$$
$$e_L = 1$$
$$w(e) = \begin{cases} \$52 & \text{if } e < 25 \\ \$100 & \text{if } e \geq 25 \end{cases} \tag{2}$$

c. Outcome 3:

$$e_H = 5$$
$$e_L = 5$$
$$w(e) = \begin{cases} \$36 & \text{if } e < 5 \\ \$52 & \text{if } e = 5 \\ \$(32 + 4e) & \text{if } e \in (5, 17) \\ \$100 & \text{if } e \geq 17 \end{cases} \tag{3}$$

d. Outcome 4. For this part, your answer should depend on the variables X and Y.

$$e_H = X$$
$$e_L = Y$$
$$w(e) = \begin{cases} \$36 & \text{if } e < Y \\ \$100 & \text{if } e \geq Y \end{cases} \tag{4}$$

Problem 4 An incumbent firm is either a low-cost type (θ_L) or a high-cost type (θ_H), each with equal probability. In period 1, the incumbent is a monopolist and sets one of two prices, p_L or p_H, with its profits in period 1 given by the following table:

Type	Profit from p_L	Profit from p_H
θ_L	6	8
θ_H	1	5

After observing the period 1 price, a potential entrant (which does not know the incumbent's type) can choose to enter the market (E) or to stay out (O) in period 2. The payoffs of both players in period 2 are as follows:

Type	Entrant's choice	Incumbent's profit	Entrant's profit
θ_L	E	0	-2
θ_L	O	8	0
θ_H	E	0	1
θ_H	O	5	0

At the beginning of the game the incumbent discounts period 2 profits using discount factor $\delta \leq 1$.

a. Draw the extensive form game. Make sure to include all payoffs, as a function of δ.

b. For the special case of $\delta = 1$, find a pooling perfect Bayesian equilibrium in which both incumbent types choose p_L in period 1. Does your equilibrium satisfy the intuitive criterion?

c. Find the range of discount factors for which a separating equilibrium exists in which type θ_L chooses p_L and type θ_H chooses p_H in period 1.

Problem 5 Consider the game in Figure 1 below.

a. Draw the reduced normal form. Find all pure strategy Nash equilibria. There is a mixed Nash equilibrium in which 1 randomizes between A and B, and 2 randomizes between L and R. Find it.

b. Find all of the game's perfect Bayesian equilibria (pure as well as mixed).

c. Explain in intuitive terms any differences between your answers to part a and part b.

Figure 1: Game for question 5