Homework 6

uncollected

Problem 1 A firm has two types of jobs, good jobs and bad jobs. When a *qualified* worker is assigned to a good job, the firm earns a net profit of $20,000. When an *unqualified* worker is assigned to a good job, the firm incurs a net loss of $20,000. When a worker of either type is assigned to a bad job, the firm breaks even. Workers prefer good jobs, and get an extra $32,000 payoff from a good job relative to a bad job.

To become qualified, a worker pays an investment cost c. This cost is higher for some workers than for others; the distribution of c across all workers is uniform between $0 and $9,000. The firm cannot observe which workers are qualified and which are not.

Suppose that while the firm cannot directly observe workers' investment decisions, it administers a test to new employees, with scores ranging from 0 to 1. The probability a qualified worker scores less than $t \in [0, 1]$ is t^2. The probability an unqualified worker scores less than t is t.

a. Suppose that the firm puts all workers with a test score of $s \in [0, 1]$ or higher into a good job. Describe the incentive constraint for a worker's decision to become qualified or not. What fraction π of workers will become qualified, as a function of s?

b. Now consider the firm's problem. Suppose that fraction π of all workers become qualified, so that the firm's prior is π. Show that the firm optimally puts workers scoring above some cutoff test score s into good jobs, and puts low-scoring workers into bad jobs, and solve for s as a function of π.

d. An equilibrium is (s, π) pair such that s maximizes firm profit given π and π is consistent with workers maximizing expected wages net of the investment cost given s. Characterize the equilibrium values of π and s as follows. One, show that $s = \frac{1}{4}$ and $\pi = \frac{2}{3}$ is an equilibrium. Two, show with a picture that there is another equilibrium with $s > \frac{1}{4}$ (you may solve for this equilibrium using a computer if you like, but the answer may not involve round numbers).

e. What economic interpretation does the Coate and Loury paper studied in class assign to the multiplicity of equilibria in its model?

Problem 2 Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains $2,000, and if an unqualified worker is assigned to the good job the employer loses $1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than θ is θ^2. The probability that an unqualified worker will have a test score less than θ is θ. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of $4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $3,000, for both A-workers and B-workers.

Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?

Problem 3 Suppose that *normal* workers have productivity of \$6, while *smart* workers have productivity of \$A, where $A > 6$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are normal, and half are smart.

Any worker can acquire as much education as she wishes, but getting e units of education costs a normal worker $B * e$, where $B > 1$, and costs a smart worker e. Assume the labor market is competitive, so that a worker earns her expected productivity. A worker's lifetime utility function is her wage minus the cost of any education she receives.

a. Suppose $A = 20$ and $B = 2$. In a graph with e on the X-axis, and wage on the Y-axis, draw 3 indifference curves for both smart and normal workers. You have enough information for your drawing to be precise.

b. Suppose $A = 20$ and $B = 2$. Construct a wage function so that there is a pooling equilibrium, with both smart and normal workers obtaining 3 units of education. Describe the wage function you chose using a graph (and, if possible, an equation).

c. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part b does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

d. Suppose that $A = 20$ and $B = 2$. Construct a wage function so that there a separating equilibrium in which normal types get education $e_N = 0$, while smart types gets $e_S = 10$. Depict the equilibrium graphically.

e. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part d does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

f. Describe, using a graph and words, the unique equilibrium outcome (e_N, e_S) of this game that satisfies the intuitive criterion.

g. For general values of A and B, determine the unique equilibrium outcome (e_N, e_S) satisfying the intuitive criterion.

h. Explain verbally how the outcome in g is affected by an increase in A. Explain intuitively why this is the case. Do the same for an increase in B.