Homework 6

answers

Problem 1 A firm has two types of jobs, good jobs and bad jobs. When a *qualified* worker is assigned to a good job, the firm earns a net profit of $20,000. When an *unqualified* worker is assigned to a good job, the firm incurs a net loss of $20,000. When a worker of either type is assigned to a bad job, the firm breaks even. Workers prefer good jobs, and get an extra $32,000 payoff from a good job relative to a bad job.

To become qualified, a worker pays an investment cost c. This cost is higher for some workers than for others; the distribution of c across all workers is uniform between $0 and $9,000. The firm cannot observe which workers are qualified and which are not.

Suppose that while the firm cannot directly observe workers' investment decisions, it administers a test to new employees, with scores ranging from 0 to 1. The probability a qualified worker scores less than $t \in [0, 1]$ is t^2. The probability an unqualified worker scores less than t is t.

a. Suppose that the firm puts all workers with a test score of $s \in [0, 1]$ or higher into a good job. Describe the incentive constraint for a worker's decision to become qualified or not. What fraction π of workers will become qualified, as a function of s?

The benefit to becoming qualified is $32,000(s - s^2)$. The cost is c. Given $c \sim U[0, 9000]$, the fraction of workers who become qualified, as a function of s, is

$$\pi = \frac{32}{9}s(1 - s) \tag{1}$$

b. Now consider the firm's problem. Suppose that fraction π of all workers become qualified, so that the firm's prior is π. Show that the firm optimally puts workers scoring above some cutoff test score s into good jobs, and puts low-scoring workers into bad jobs, and solve for s as a function of π.

The firm's posterior belief that a given worker who received test score θ is qualified is $p(\theta) = \frac{\pi 2\theta}{\pi 2\theta + 1 - \pi}$. The firm will put the worker into the good job iff $p * 20,000 - (1 - p)20,000 \geq 0$. Simplifying, the firm will put a worker into a good job iff $\pi \geq \frac{1}{1+2\theta}$. The firm's cutoff test score is determined by where this holds with equality, or

$$s = \frac{1 - \pi}{2\pi} \tag{2}$$

d. An equilibrium is (s, π) pair such that s maximizes firm profit given π and π is consistent with workers maximizing expected wages net of the investment cost given s. Characterize the equilibrium values of π and s as follows. One, show that $s = \frac{1}{4}$ and $\pi = \frac{2}{3}$ is an equilibrium. Two, show with a picture that there is another equilibrium with $s > \frac{1}{4}$ (you may solve for this equilibrium using a computer if you like, but the answer may not involve round numbers).

The pair $(\pi, s) = (\frac{2}{3}, \frac{1}{4})$ clearly satisfies both (1) and (2), as required for an equilibrium. A picture helps to show that there is a second equilibrium. See the figure at the end of this answer set. Equation (1) is a concave function maximizes at $s = \frac{1}{2}$ and equal to zero at $s = 0$ and $s = 1$. Equation (2) is a downward sloping function that crosses the red line at $s = \frac{1}{4}$ and is always positive. This implies that there must be a second equilibrium at $s_2^* > \frac{1}{4}$.

e. What economic interpretation does the Coate and Loury paper studied in class assign to the multiplicity of equilibria in its model?

The possibility of rational discrimination; "bad" equilibria correspond to discriminated-against groups, and "good" equilibria correspond to favored groups.

Problem 2 Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$2,000, and if an unqualified worker is assigned to the good job the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than θ is θ^2. The probability that an unqualified worker will have a test score less than θ is θ. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of \$4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and \$3,000, for both A-workers and B-workers.

Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?

Using the notation from class, we have:

$$
\begin{aligned}
x_q &= 2000 \\
x_u &= 1000 \\
F_q(\theta) &= \theta^2 \\
f_q(\theta) &= 2\theta \\
F_u(\theta) &= \theta \\
f_u(\theta) &= 1 \\
\omega &= 4000 \\
G(c) &= \frac{c}{3000} \quad \text{for } c \in [0, 3000]
\end{aligned}
$$

The EE and WW equations are then given by:

$$
\begin{aligned}
2 &= \frac{1-\pi}{\pi}\frac{1}{2s} \quad \text{(EE)} \\
\pi &= \frac{4}{3}s(1-s) \quad \text{(WW)}
\end{aligned}
$$

Solving (EE) and (WW) for π and s yields two solutions in which both π and s are in $[0, 1]$:

$$\text{Solution 1: } s = \frac{3}{4}, \pi = \frac{1}{4}$$
$$\text{Solution 2: } s = \frac{1}{2}, \pi = \frac{1}{3}$$

If the ($s = \frac{3}{4}, \pi = 14$) equilibrium is applied to B-workers, while the ($s = \frac{1}{2}, \pi = 13$) equilibrium is applied to A-workers, then, despite A- and B-workers being ex ante identical, statistical discrimination against B-workers will persist in equilibrium.

Problem 3 Suppose that *normal* workers have productivity of \$6, while *smart* workers have productivity of \$A, where $A > 6$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are normal, and half are smart.

Any worker can acquire as much education as she wishes, but getting e units of education costs a normal worker $B * e$, where $B > 1$, and costs a smart worker e. Assume the labor market is competitive, so that a worker earns her expected productivity. A worker's lifetime utility function is her wage minus the cost of any education she receives.

a. Suppose $A = 20$ and $B = 2$. In a graph with e on the X-axis, and wage on the Y-axis, draw 3 indifference curves for both smart and normal workers. You have enough information for your drawing to be precise.

All graphs appear at the end of this answer sheet. Note that a smart worker's utility is $wage - e$, and so the equation for the indifference curve giving her (say) utility of 20 is $wage - e = 20$. Since we will graph this curve with wage on the y-axis and e on the x-axis, solve for wage: $wage = 20 + e$. The equation for an indifference curve for utility 10 would be $wage = 10 + e$, and so on.

b. Suppose $A = 20$ and $B = 2$. Construct a wage function so that there is a pooling equilibrium, with both smart and normal workers obtaining 3 units of education. Describe the wage function you chose using a graph (and, if possible, an equation).

One wage function that would support these education levels as a pooling equilibrium is the following:

$$wage(e) = \begin{cases} 13 & \text{if } e = 3 \\ 6 & \text{if } e \neq 3 \end{cases} \tag{3}$$

See the end of the answer sheet for a picture.

c. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part b does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

Consider a worker who gets education $e = 10$. Any wage function which supports ($e_L = e_H = 3$) as a pooling equilibrium must put at least some weight on a worker with $e = 10$ being a low type, as otherwise the high type of worker would surely prefer to switch from $e = 12$ to $e = 10$ (her utility would increase from $8to10$). However, low types can only be made worse off by choosing $e = 10$, no matter what the wage is. Even if a low type is paid the maximum wage of 20 (the productivity of a high type), his utility would be only $20 - 2 * 10 = 0$, whereas he gets utility 5 from choosing 3 units of education. Therefore, the intuitive criterion requires firms to hold belief $\mu(H|e = 10) = 1$, in which case they must pay a wage of 20 for any worker who chooses 10 units of education. This breaks the equilibrium identified in part b.

d. Suppose that $A = 20$ and $B = 2$. Construct a wage function so that there a separating equilibrium in which normal types get education $e_N = 0$, while smart types gets $e_S = 10$. Depict the equilibrium graphically.

One wage function that would support these education levels as a separating equilibrium is the following:

$$wage(e) = \begin{cases} 20 & \text{if } e = 10 \\ 6 & \text{if } e \neq 10 \end{cases} \tag{4}$$

See the end of the answer sheet for a picture.

e. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part d does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

Consider a worker who chooses $e = 8$. Any wage function that is part of an equilibrium must pay $wage(8) < 20$, otherwise the high type of worker would surely choose to switch to $e = 8$. But the intuitive criterion says that a worker choosing $e = 8$ must be a high type, since the low type of worker could only be made worse off relative to his equilibrium payoff by choosing $e = 8$. The low type gets utility of 6 in the equilibrium of part d, yet even if he were paid the maximum wage of 20, would get utility of only 4 from choosing $e = 8$. Therefore, the intuitive criterion requires $wage(e = 8) = 20$, which breaks the equilibrium in part d.

f. Describe, using a graph and words, the unique equilibrium outcome (e_N, e_S) of this game that satisfies the intuitive criterion.

The unique equilibrium satisfying the intuitive criterion is a separating equilibrium where the high types get just enough education to leave the low types indifferent between switching to e_H and staying at $e_L = 0$. Since a choice of $e = 0$ gives a low type utility 6, a choice of $e = 7$ and a wage of 20 would give the low type the same utility. Hence, the unique equilibrium outcome is $e_L = 0, e_H = 7$, and this is supported by a wage function such as the following:

$$wage(e) = \begin{cases} 20 & \text{if } e = 7 \\ 6 & \text{if } e \neq 7 \end{cases} \tag{5}$$

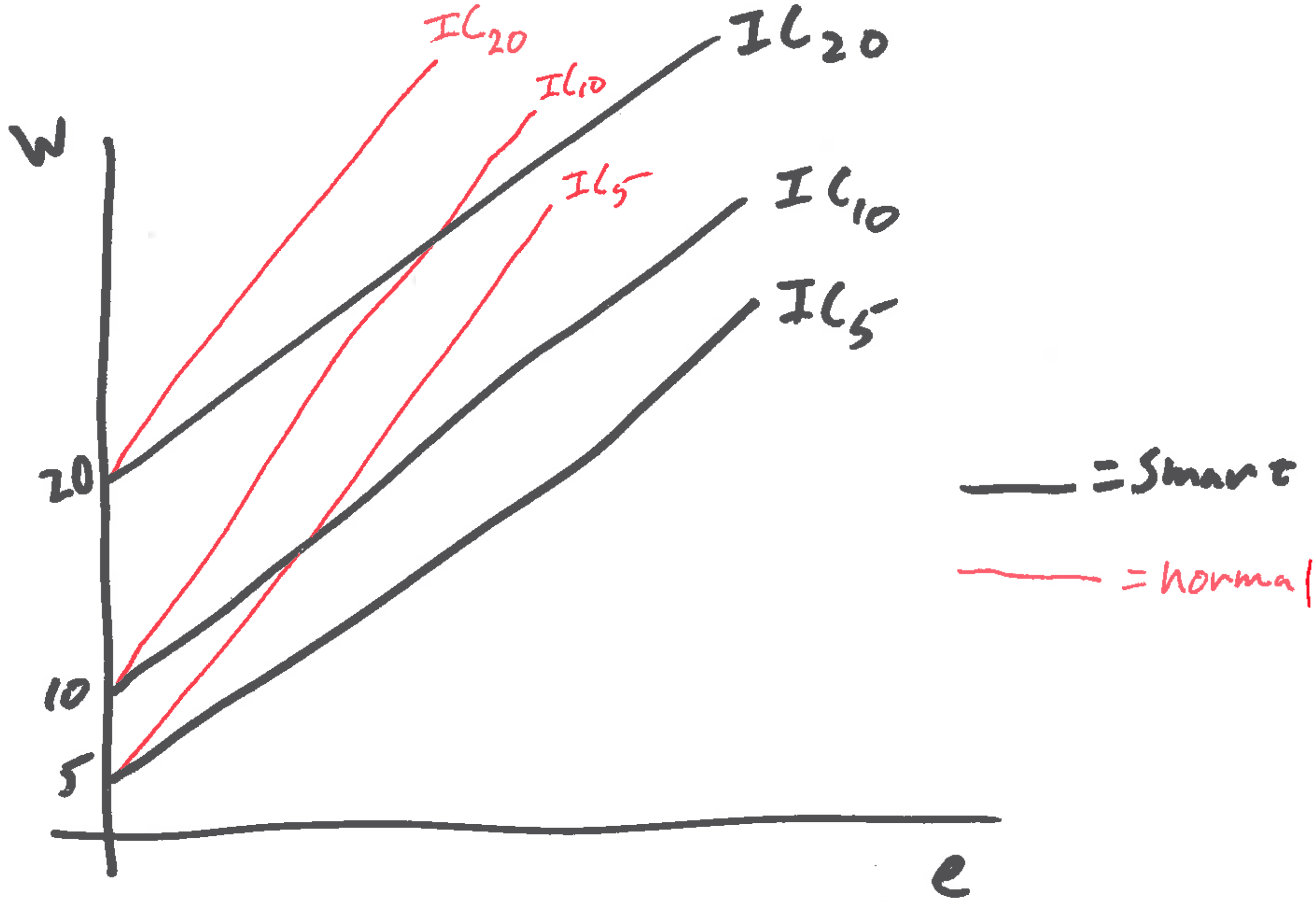
g. For general values of A and B, determine the unique equilibrium outcome (e_N, e_S) satisfying the intuitive criterion.

To leave the low type indifferent between switching and not, e_H must satisfy $6 = A - be_H$, so $e_H = \frac{A-6}{b}$.

h. Explain verbally how the outcome in g is affected by an increase in A. Explain intuitively why this is the case. Do the same for an increase in B.

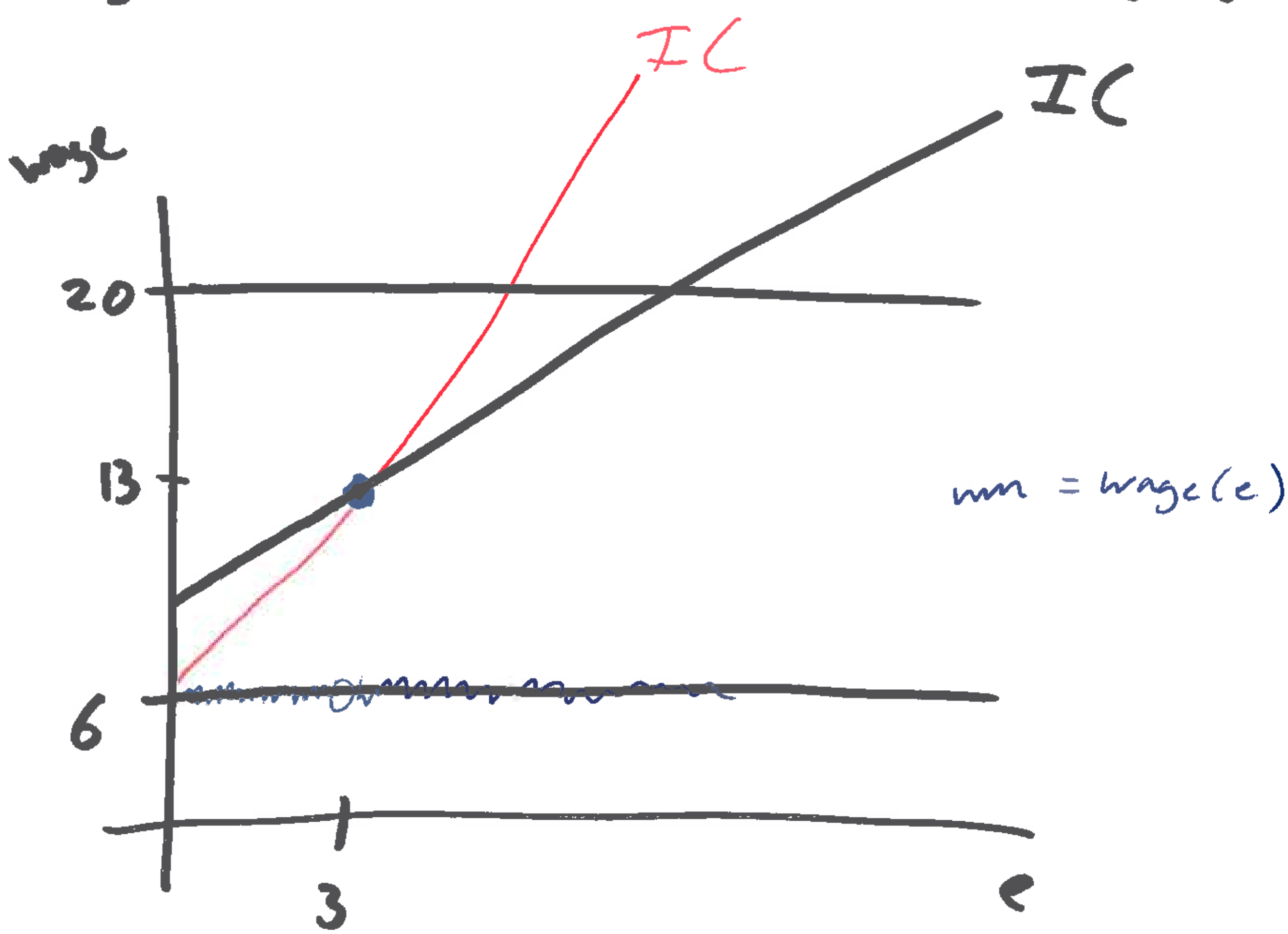
e_H is increasing in A and decreasing in b. The intuition is that as A increases, imitating a high type becomes relatively more valuable, and so high types must choose a higher level of education to deter the low types from choosing e_H. As b increases, education becomes more costly for the low types, and so the high types do not need to choose as high of a level of e_H to deter the low types from imitating the high types.

1a

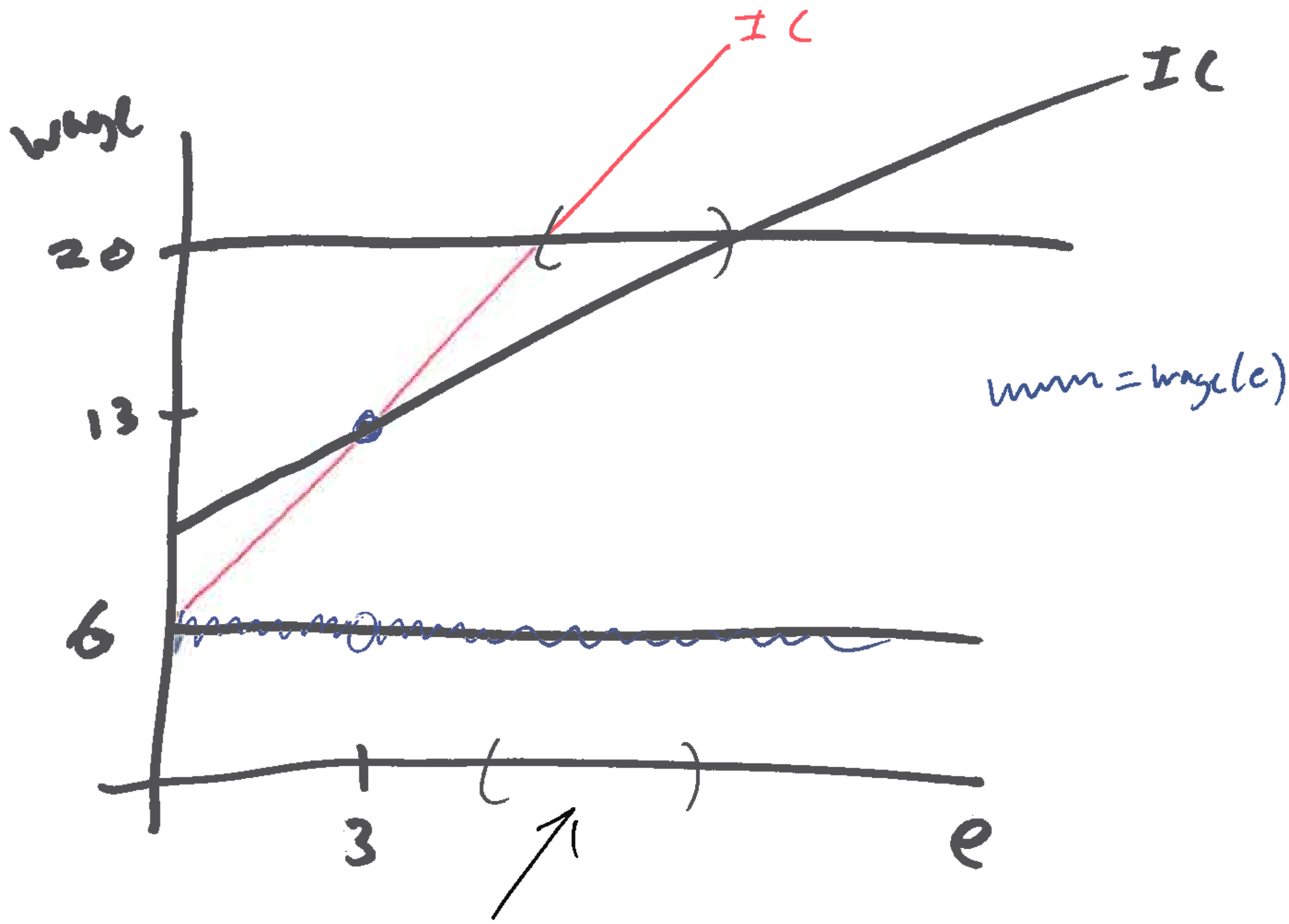


Smart workers' indifference curves

1b.

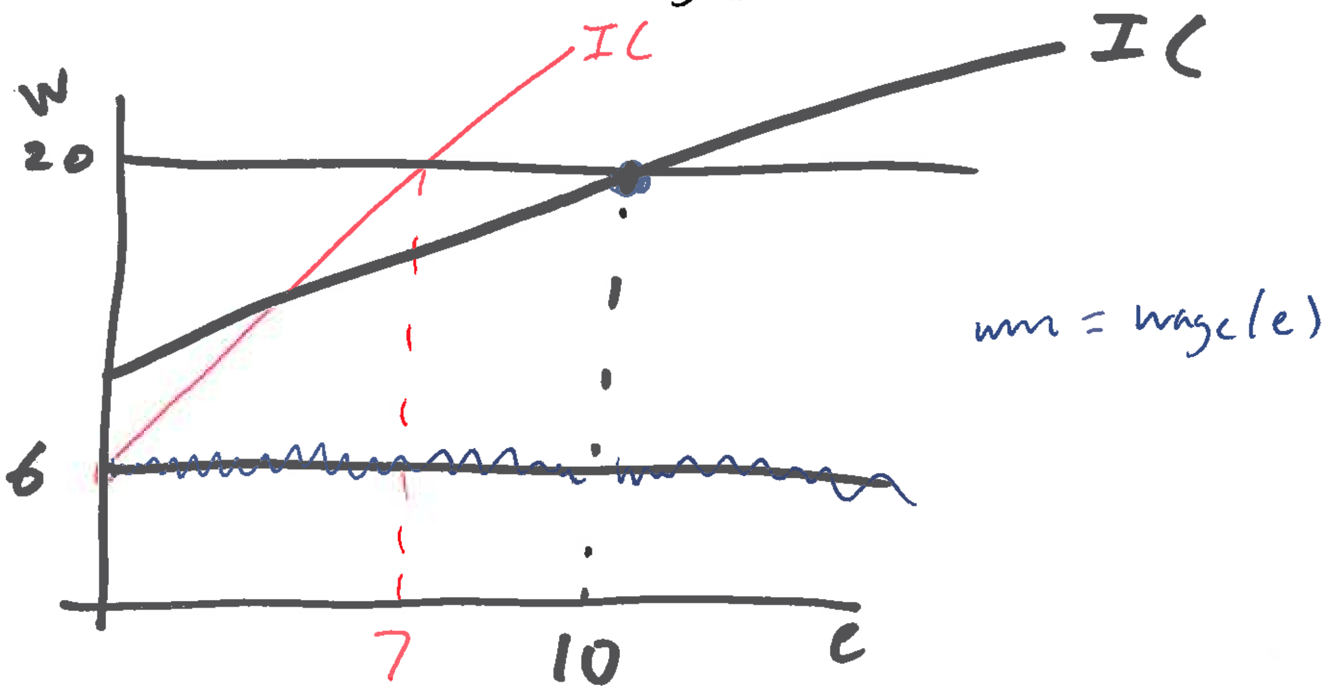


1c.

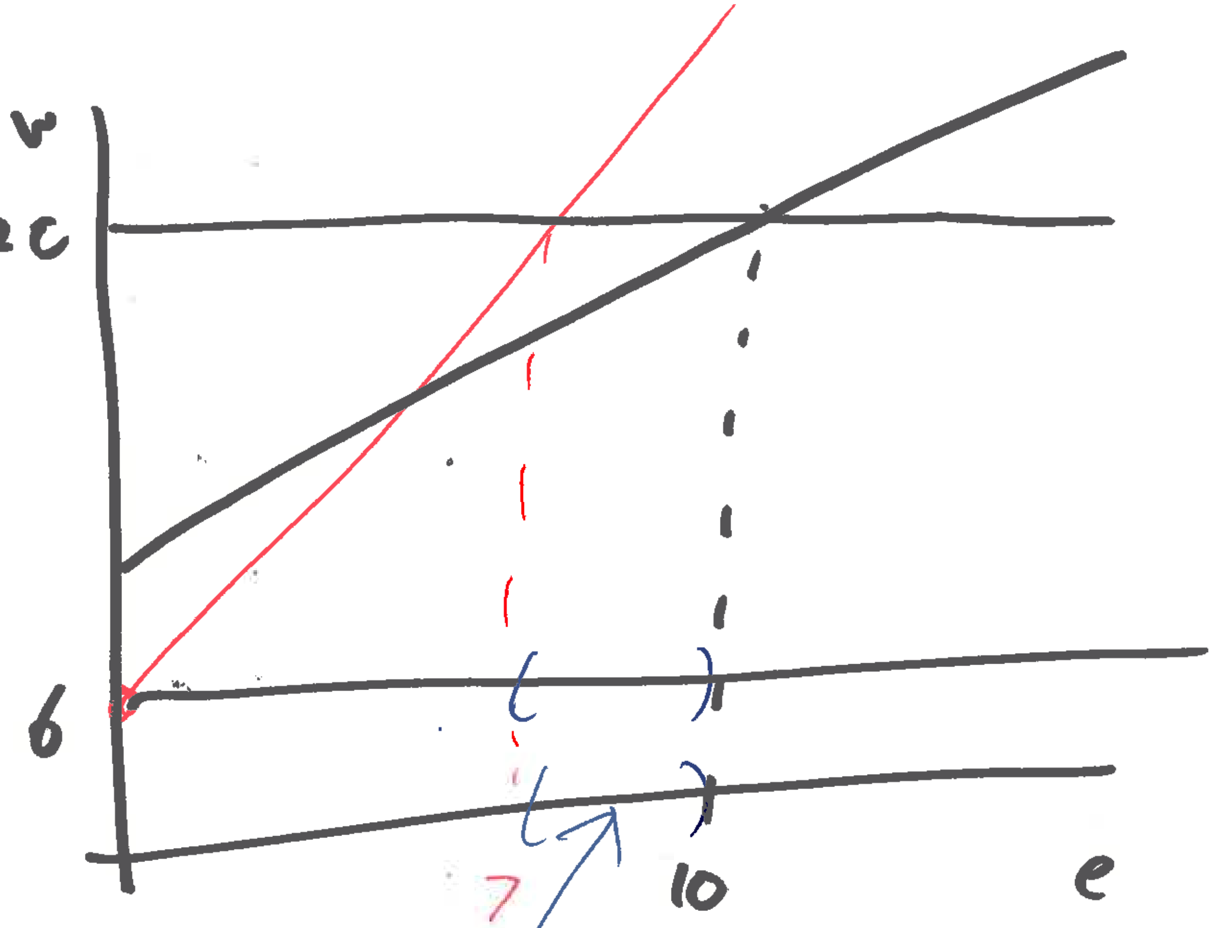


For any education level in this range, the firm should pay a wage of 20, as only the ~~hig~~ smart workers would possibly choose a value of e in this range.

1d.

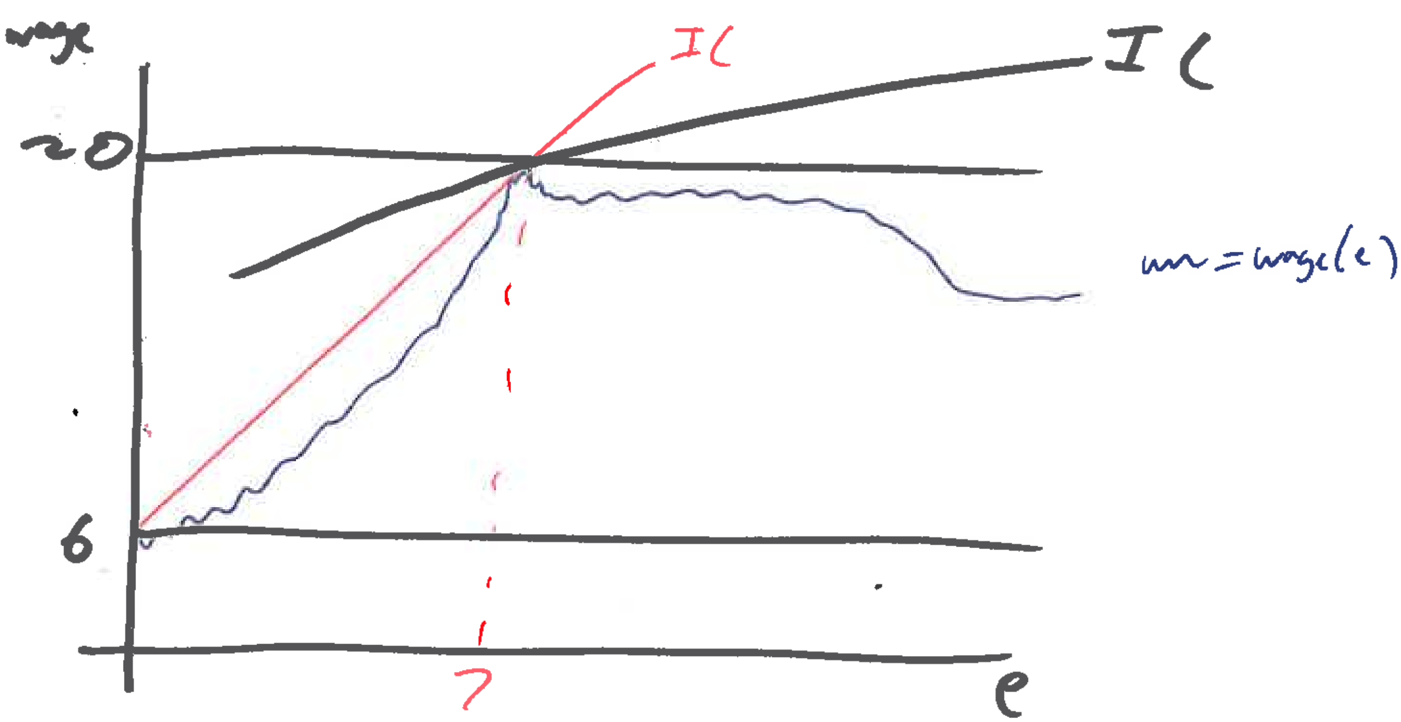


1e



for any education level between 7 and 10, the firm should recognize that only the high smart type is potentially better off, and thus should pay a wage of 20.

1f.



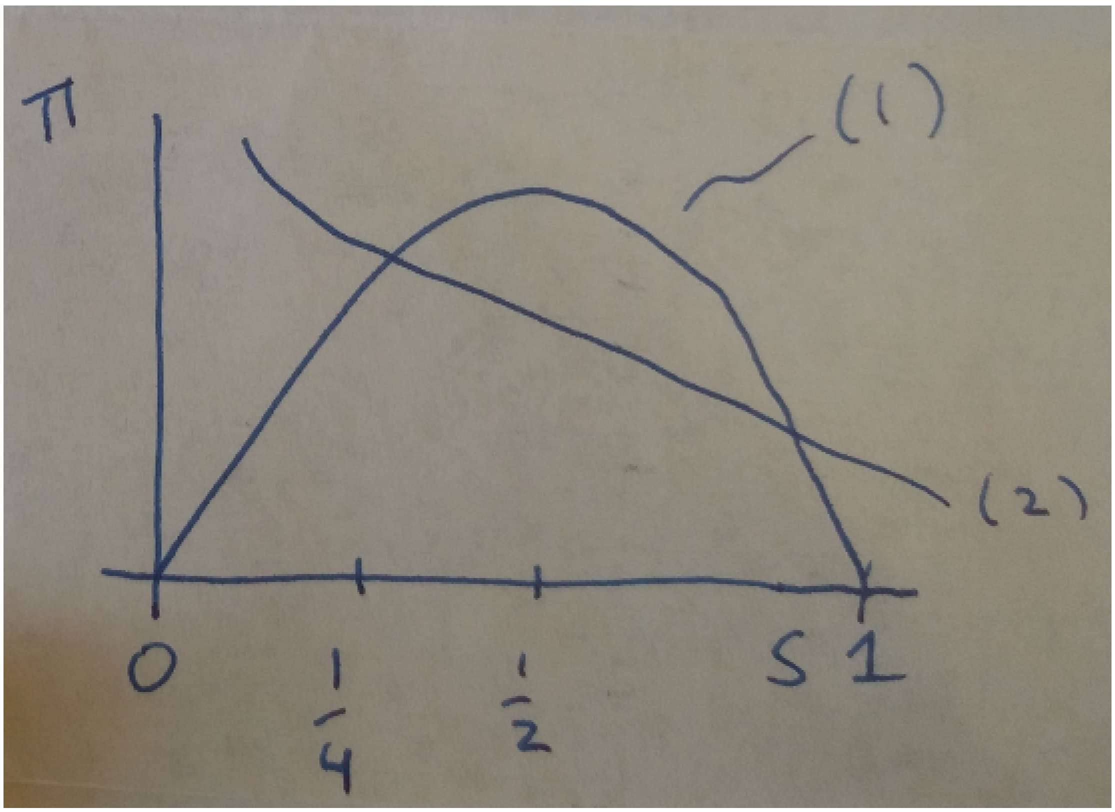


Figure 1: Figure mentioned in the answer to 1d.