Midterm

2/23/2017

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup, an intuitive explanation, or the right approach demonstrated on a simplified version of the problem. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 120 minutes to complete the exam. Good luck!

Problem 1 (15 points) Refer to the Games 1 and 2 depicted in Figures 1 and 2.

a. Enter payoffs into the games below so that they have the following properties:

- The games are equivalent except for the addition of strategy C to game 2 (i.e. the first two rows of Game 1 are equivalent to the first two rows of Game 2).
- The only Nash equilibrium of Game 1 is (A, L).
- The only Nash equilibrium of Game 2 is (C, R).

Please copy the games onto your answer sheets to complete this question.

b. Referring only to Game 1, enter payoffs such that the only Nash equilibrium is both players mixing with equal probability on each strategy.

Player 1 \ Player 2	L	R
A	3,3	
B		

Figure 1: Game 1

Player 1 \ Player 2	L	R
A	3,3	
B		
C		2,2

Figure 2: Game 2

Problem 2 (15 points) In 1897, Sears and Montgomery Ward operated two competing mail-order catalogs.

Suppose that demand, in thousands of units, for a certain type of coal-powered stove[1] sold in both catalogs, but not easily obtainable elsewhere, is given by $Q = 20 - \min\{P_S, P_M\}$. The firm with the lower catalog price supplies the entire market, while if prices are equal, the two firms each supply half of the market. Suppose that Sears and Montgomery Ward have identical marginal costs of \$10, which include all acquisition, storage, and shipping costs.

a. Sears and Montgomery Ward both release their catalogs on the same day, which means neither is aware of the other's price at the time it sets its own. What is the equilibrium price set by each catalog?

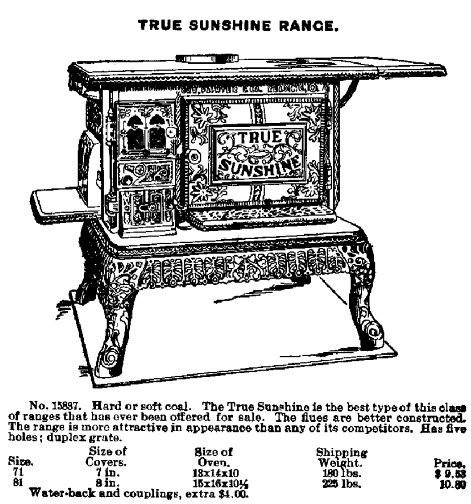
b. Now suppose that in 1898, Montgomery Ward releases its calendar on January 1, but Sears releases its calendar on July 1, so that it can observe Montgomery Ward's posted price on this item before setting its own price. Would this change the equilibrium outcome from part a.?

Modern internet retailers like Amazon scrape rivals' websites to continually track their prices; often, the scraping algorithm is empowered to automatically match rivals' prices. In game theoretic terms, Amazon is able to credibly commit to a strategy of matching its rivals' prices.

c. Suppose that amazon.com and walmart.com are duopoly sellers of a certain item,[2] with demand in thousands of units given by $Q = 20 - \min\{P_A, P_W\}$. Suppose that each has a marginal cost of \$10. Suppose that Wal-Mart believes that Amazon has committed to match its price exactly, whatever that price is. What price should Wal-Mart set?

d. Given your answer to part c., is amazon.com better off programming its algorithm to match walmart.com's price exactly, or to undercut it by 1 cent?

[1]Specifically, the True Sunshine Range, with oven dimensions 15x16x10$\frac{1}{2}$ and a shipping weight of 225 pounds.
[2]Specifically, bandages that look like bacon strips.


TRUE SUNSHINE RANGE.

No. 15887. Hard or soft coal. The True Sunshine is the best type of this class of ranges that has ever been offered for sale. The flues are better constructed. The range is more attractive in appearance than any of its competitors. Has five holes; duplex grate.

Size.	Size of Covers.	Size of Oven.	Shipping Weight.	Price.
71	7 in.	13x14x10	180 lbs.	\$ 9.53
81	8 in.	15x16x10½	225 lbs.	10.80

Water-back and couplings, extra \$4.00.

Problem 3 (15 points) Consider a market with demand curve $P = 1 - Q$, served by Cournot oligopolists with marginal cost c who compete by simultaneously setting quantity.

a. Suppose that there are 2 firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

b. Now suppose that there are N firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

Problem 4 (15 points) This problem refers to the following game:

	A	B	C
A	8,8	0,10	-2,0
B	10,0	2,2	0,0
C	0,-2	0,0	2,2

a. What are the pure strategy Nash equilibria?

b. Is there a mixed strategy Nash equilibrium where both players mix A and B? If so, find the equilibrium. If not, explain why not.

c. Is there a mixed strategy Nash equilibrium where both players mix B and C? If so, find the equilibrium. If not, explain why not.

Problem 5 (15 points) Consider the following game:

		Player 2		
		X	Y	Z
	A	3,3	-2,0	0,1
Player 1	B	5,-5	0,0	0,0
	C	0,0	$\frac{1}{2}$,0	1,1

a. Solve for all Nash equilibria, pure and mixed.

b. Now suppose that this game is repeatedly infinitely often, and that both players have discount factor δ. For what values of δ do the following strategies comprise a subgame perfect equilibrium of the repeated game?

Phase I: Play (A, X) initially, and if (A, X) was played in the previous period. If either player deviates from (A, X) permanently move to Phase II in the following period.

Phase II: Play (C, Z).

c. Again suppose that the game is played repeatedly, but this time determine the minimum value of δ for which the following strategies comprise a subgame perfect equilibrium:

Phase I: Play (A, X) initially, and if (A, X) was played in the previous period. If either player deviates from (A, X) switch to Phase II in the following period.

Phase II: Play (B, Y) for 2 periods, and then switch to Phase I. If either player deviates from (B, Y), begin Phase II again.

Problem 6 (15 points) Consider a 2-player bargaining game, in which the players are choosing how to split a surplus of \$1. Each player discounts payoffs one period in the future by $\delta = \frac{1}{2}$. The rules are as follows:

Period 0: Player 1 chooses a value of X between 0 and 1, at cost $c(X) = \frac{3}{8}X^2$.

Period 1: Player 2 makes an offer of $(y, 1-y)$, where y is between 0 and 1. Player 1 may either accept the offer, in which case the surplus is split accordingly and the game ends, or reject the offer, in which case the game moves on to period 2.

Period 2: Player 1 makes an offer to player 2. If the offer is accepted, the surplus is split accordingly and the game ends. If the offer is rejected, a mediator awards a payoff of X to player 1, and $1 - X$ to player 2 (nb. this is the same X chosen by player 1 in period 0; interpret the cost player 1 incurred in that period as that of hiring an attorney to investigate the most favorable mediation venue).

a. Solve for the subgame perfect equilibrium of this game. Is player 1 or player 2 better off in this setup?

b. Intuitively, how would your answer change by an increase in δ? You do not need to work out the mathematical details.