Homework 1

due 1/25/18

Problem 1 Kirt and Lila are engaged in a joint project. If person $i \in \{K, L\}$ invests effort $x_i \in [0,1]$ in the project, at cost $c(x_i)$, the outcome of the project is worth $f(x_K, x_L)$. The worth of the project is split equally by Kirt and Lila, regardless of their effort levels, so that each gets a payoff of $\frac{1}{2}f(x_K, x_L) - c(x_i)$. Suppose effort levels are chosen simultaneously.

a. Suppose $f(x_K, x_L) = 3x_Kx_L$ and that $c(x_i) = x_i^2$. Find the Nash equilibrium effort levels of this simultaneous move game.

b. Is there a pair of effort levels that yield higher payoffs for both players than do the Nash equilibrium effort levels in part a.?

Problem 2 Consider the normal form game below:

Avon

Joe	I	N
I	r, r	$r-1, 0$
N	$0, r-1$	$0, 0$

In this game, strategy I represents investing, and strategy N represents not investing. Investing yields a payoff of r or $r-1$, according to whether the player's opponent invests or not. Not investing yields a certain payoff of 0.

Describe the set of Nash equilibria (pure and mixed) of the game for each $r \in [-2, 3]$.

Problem 3 Gibbons, problem 1.13

Problem 4 This problem refers to the following game:

	A	B	C
A	4,4	0,5	-1,0
B	5,0	1,1	0,0
C	0,-1	0,0	1,1

a. What are the pure-strategy Nash equilibria?

b. Is there a mixed strategy Nash equilibrium where both players mix A and B? If so, find the equilibrium. If not, explain why not.

c. Is there a mixed-strategy Nash equilibrium where both players mix B and C? If so, find the equilibrium. If not, explain why not.

Problem 5 In the game below, which strategies survive iterated removal of strictly dominated strategies? What are the pure strategy Nash equilibria?

	L	C	R
T	1,3	5,4	4,2
M	2,3	3,1	3,2
B	3,5	4,7	1,4

Problem 6 99 shepherds share a common field in which they graze their sheep. Each shepherd purchases as many sheep as he/she likes, at a cost of $c = \$300$/sheep. The value of one sheep is given by:

$$v(G) = 2000 - S$$

where S is the total number of sheep which graze in the field (more sheep mean less grass/sheep, more sheep fights, etc). The common field is the only suitable location for grazing, and sheep die without grazing, so you may assume that all purchased sheep are brought to graze in the field.

a. In a symmetric Nash equilibrium, how many sheep does each shepherd purchase? How much profit is earned by each shepherd?

b. What is the socially optimal number of sheep (i.e., the number of sheep that would maximize total profit of all 99 shepherds)? What is the profit for each shepherd if this number of sheep is ?

c. Suppose a government imposes a tax on sheep of \$T/head, but that the revenue collected from the tax is distributed evenly to each of the 99 shepherds, regardless of how many sheep the shepherd owns. Could such a tax be welfare enhancing? Why or why not?

Problem 7 Refer to the Games 1 and 2 depicted in Figures 1 and 2.

a. Enter payoffs into the games below so that they have each of the following properties:

- The games are equivalent except for the addition of strategy C to game 2 (i.e. the first two rows of Game 1 are equivalent to the first two rows of Game 2).
- The only Nash equilibrium of Game 1 is (A, L).
- The only Nash equilibrium of Game 2 is (C, R).

b. Referring only to Game 1, enter payoffs such that the only Nash equilibrium is both players mixing with equal probability on each strategy.

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Player 2

Player 1	L	R
A	3,3	
B		

Figure 1: Game 1

Player 2

Player 1	L	R
A	3,3	
B		
C		2,2

Figure 2: Game 2