Homework 3

due by 11:59pm, 2/27/18

Problem 1 Consider the game below:

	Cooperate	Defect
Cooperate	4,4	0,12
Defect	12,0	2,2

Suppose the game is played repeatedly, with both players sharing a discount factor $\delta \in (0,1)$. For what range of values of δ do grim trigger strategies (cooperate so long as no one has defected, otherwise defect forever) comprise a subgame perfect equilibrium?

Problem 2 The inverse market demand curve for a good is given by $P = 1 - Q$. Two Cournot competitors compete to supply the market ($Q = q_1 + q_2$), each with zero marginal and fixed costs.

a. If the two firms were to collude to make joint profits as large as possible, what quantities would they produce, and what profit would they earn? You may assume that they each produce the same quantity.

b. Suppose one of the firms is considering unilaterally deviating from the collusive outcome in a. What is the maximal profit the firm can attain with such a deviation?

c. Suppose the oligopoly stage game is repeated infinitely often, and that both firms share discount factor δ. For what range of δ can the firms sustain the collusive outcome identified in a, using grim trigger strategies with the Nash equilibrium as the punishment?

Problem 3 Consider the following game:

	L	C	R
U	1,1	21,0	0,0
M	0,21	4,4	0,1
D	0,0	1,0	-1,-1

Suppose the game is repeated infinitely often, with both players sharing discount factor $\delta = .9$. Both players play limited punishment trigger strategies with a punishment phase of T periods of the Nash equilibrium outcome (U,L). Determine the length of the punishment period that is required to support $(4,4)$ as the payoff in every stage of a subgame perfect equilibrium (how small can we make T such that no player finds it optimal to deviate in any period?).

Problem 4 Consider an industry that consists of two firms, A and B. They face a demand curve $Q = q_A + q_B = 14 - P$, where P is the industry price of output. Both firms have constant marginal cost of \$2.

a. Suppose they form a cartel and choose the price that maximizes the sum of their profits. Show that they will choose P = \$8.

b. Now suppose that instead of forming a cartel, they choose prices simultaneously. If they choose different prices, the firm that chooses the lower price captures the entire market; if they set the same price they split the market evenly. Suppose they play this game once. Show that in a Bertrand equilibrium, both firms will charge \$2.

c. Suppose they play this game an infinite number of times. Consider the following grim trigger strategy. Choose the cartel price (i.e., P = \$8) in the first period. Continue to choose the cartel price in subsequent periods if all firms have always chose the cartel price up to that point. If any firm chose a price other than \$8, choose the Bertrand price (i.e., \$2) from that point forward.

For what range of values of the discount factor do these trigger strategies constitute a subgame perfect equilibrium?

d. Now change this game so that there are $N \geq 2$ oligopolists; thus if they all charge the same price, each will sell a proportion $\frac{1}{N}$ of the market demand at that price. Express the critical discount factor (required to obtain an equilibrium in the infinitely repeated game) as a function of N. Does your answer suggest that it will be easier to sustain cooperation when N is small or when N is large? What is the intuition behind this result?