Homework 4
due March 29, 2018

Problem 1 A seller has a painting for sale that is either good or bad. A good painting is worth 1 to the seller. A bad painting is worth 0 to the seller. The seller knows the painting's quality. The buyer does not know whether the painting is good or bad, only that it is good with probability $\frac{1}{2}$ and bad with probability $\frac{1}{2}$. A good painting is worth v to the buyer. A bad painting is worth 0 to the buyer.

The buyer makes a one-time offer to the seller, which the seller can accept or reject. To keep the problem simple, assume that the seller accepts offers where she is indifferent.

a.   Suppose $v = 1$. What offer should the buyer make? What is his expected profit?

b.   Suppose $v = 1.5$. What offer should the buyer make? What is his expected profit?

c.   Suppose $v = 5$. What offer should the buyer make? What is his expected profit?

d.   What is the lowest value of v such that both types of the painting are traded in equilibrium?

e.   Discuss the efficiency of the outcome in a., b. and c. What is the source of the inefficiency, if any?

Problem 2 The best available test for Groat's disease is pretty accurate, but sometimes returns false positives and false negatives. Specifically, the test returns a positive result for 92 out of every 100 individuals with Groat's disease (and a negative test result for 8 out of 100 individuals who have Groat's). The test returns a negative result for 99 out of every 100 individuals who do not have Groat's disease (and a positive test result for 1 out of every 100 individuals who do not have Groat's). Only one in ten thousand people has Groat's disease.

a.   Suppose that Alice receives a positive test result. What is the probability that Alice has Groat's? You may assume that prior to taking the test Alice's probability of having Groat's was one in ten thousand.

b.   Suppose a new test is developed which has no false negatives; everyone with Groat's who takes the test receives a positive result. It is still the case that the test returns a negative result for 99 out of 100 people who do not have Groat's. Suppose that Bob takes this new test, and receives a positive result. What is the probability that Bob has Groat's? Again, you may assume that Bob's prior probability of having Groat's is one in ten thousand.

c.   What can you conclude about the effectiveness of preventative screening for rare diseases?

Problem 3 Consider a two-player Bayesian game where both players are not sure whether they are playing game X or game Y, and they both think that the two games are equally likely. This game has a unique Bayesian Nash equilibrium, which involves only pure strategies. What is it? (Hint: start by looking for Player 2's best response to each of Player 1's actions.)

Player 2

		L	M	R	
	T	1,.2	1,0	1,.3	Game X
Player 1	B	2,2	0,0	0,3	

Player 2

	L	M	R
T	1,.2	1,.3	1,0
B	2,2	0,3	0,0

Player 1   Game Y

Problem 4 Now consider a variant of this game (from Problem 2) in which Player 2 knows which game is being played (but Player 1 still does not). This game also has a unique Bayesian Nash equilibrium. What is it? (Hint: Player 2's strategy must specify what she chooses in the case that the game is X and in the case that it is Y.) Compare Player 2's payoff in the games from Problems 2 and 3. What seems strange about this?

Problem 5 Firm 1 is considering taking over Firm 2. It does not know Firm 2's current value, but believes that is equally likely to be any dollar amount from 0 to 100. If Firm 1 takes over firm 2, it will be worth 50% more than its current value, which Firm 2 knows to be x. Firm 1 can bid any amount y to take over Firm 2 and Firm 2 can accept or reject this offer. If 2 accepts 1's offer, 1's payoff is $\frac{3}{2}x - y$, and 2's payoff is y. If 2 rejects 1's offer, 1's payoff is 0 and 2's payoff is x.

a. Find the unique Bayesian Nash equilibrium of this game.

b. Can you explain why the result you obtained in part a is sometimes called "adverse selection"? Give two other examples of markets that may exhibit adverse selection.

Problem 6 Two bidders are bidding on a bottle of Scotch in a first-price, sealed bid auction. Bidder 1 values the bottle at v_1, and bidder 2 values the bottle at v_2. Neither bidder knows the other's valuation, but each knows that $v_i \sim U[0,1]$, and that v_1 and v_2 are independent (note that this setting is identical to the first example studied in class). Bidders simultaneously submit hidden bids; the highest bidder gets the bottle for the price he paid.

a. Show that there is a Nash equilibrium in bidding strategies in which player i bids $b_i = \frac{v_i}{2}$.

b. Suppose that bidder 2 is irrational, and will bid $b_2 = v_2$. Demonstrate that $b_1 = \frac{v_1}{2}$ remains the best response for player 1.

c. Suppose a third bidder arrives to bid on the bottle of Scotch. Like bidders 1 and 2, bidder 3's valuation is private information, but is distributed $v_3 \sim U[0,1]$. v_1, v_2, and v_3 are independent. Show that there is a Nash equilibrium in which player i bids $b_i = \frac{2v_1}{3}$.

To answer part c., you will need to use the fact that if X_1 and X_2 are independent $U[0,1]$ random variables, $P(\max\{X_1, X_2\} < x) = x^2$ for $x \in [0,1]$.

Problem 7 Consider two Cournot oligopolists, facing demand curve $P = 1 - q_1 - q_2$. Firm 1's marginal cost is as follows:

$$c_1 = 0 \text{ w.p. } (1 - \alpha) \text{ (Firm 1 is low cost)}$$
$$c_1 = X \text{ w.p. } \alpha \text{ (Firm 1 is high cost)}$$

Firm 1 knows its marginal cost, but Firm 2 knows only the distribution given above. Firm 2 has marginal cost equal to 0.

a. Solve for Firm 1's best response functions. Note that since there are two types of Firm 1 (high and low cost), Firm 1 has two best response functions.

b. Solve for Firm 2's best response function.

c. In the oligopoly game's Bayesian Nash equilibrium, what quantity does Firm 2 produce? What quantity does Firm 1 produce if its costs are low? If its costs are high? What is the market price in each case?

d. What is the derivative of q_1 with respect to X in the case that Firm 1 is a low cost firm? In the case Firm 1 is high cost? Why does Firm 1's quantity depend on X even in the former case?