Homework 5

uncollected

Problem 1 Suppose that *normal* workers have productivity of \$6, while *smart* workers have productivity of \$A, where $A > 6$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are normal, and half are smart.

Any worker can acquire as much education as she wishes, but getting e units of education costs a normal worker $B * e$, where $B > 1$, and costs a smart worker e. Assume the labor market is competitive, so that a worker earns her expected productivity. A worker's lifetime utility function is her wage minus the cost of any education she receives.

a. Suppose $A = 20$ and $B = 2$. In a graph with e on the X-axis, and wage on the Y-axis, draw 3 indifference curves for both smart and normal workers. You have enough information for your drawing to be precise.

b. Suppose $A = 20$ and $B = 2$. Construct a wage function so that there is a pooling equilibrium, with both smart and normal workers obtaining 3 units of education. Describe the wage function you chose using a graph (and, if possible, an equation).

c. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part b does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

d. Suppose that $A = 20$ and $B = 2$. Construct a wage function so that there a separating equilibrium in which normal types get education $e_N = 0$, while smart types gets $e_S = 10$. Depict the equilibrium graphically.

e. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part d does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

f. Describe, using a graph and words, the unique equilibrium outcome (e_N, e_S) of this game that satisfies the intuitive criterion.

g. For general values of A and B, determine the unique equilibrium outcome (e_N, e_S) satisfying the intuitive criterion.

h. Explain verbally how the outcome in g is affected by an increase in A. Explain intuitively why this is the case. Do the same for an increase in B.

Problem 2 Suppose that *low-ability* workers have productivity of D, while *high-ability* workers have productivity of A, where $A > D$. Firms cannot tell low-ability workers from high-ability workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are low-ability, and half are high-ability.

Any worker can acquire as much education as she wishes, but getting e units of education costs a low-ability worker $B*e$, where $B > 1$, and costs a high-ability worker e. Assume the labor market is competitive, so that a worker earns her expected productivity.

a. Suppose $A = 15$, $B = 4$, and $D = 1$. Does there exist a pooling equilibrium in which both high- and low-ability workers get 1 unit of education? If so, draw or describe a wage function that supports this equilibrium outcome, and determine whether or not the equilibrium satisfies the intuitive criterion. If not, explain why not.

b. Suppose $A = 15$, $B = 4$, and $D = 1$. Does there exist a pooling equilibrium in which both high- and low-ability workers get 3 units of education? If so, draw or describe a wage function that supports this equilibrium outcome, and determine whether or not the equilibrium satisfies the intuitive criterion. If not, explain why not.

c. Suppose $A = 15$, $B = 4$, and $D = 1$. Solve for a separating equilibrium which *does not* satisfy the intuitive criterion. Draw or describe a wage function that supports this outcome in an equilibrium. Clearly explain, using your graph and/or a verbal description, why this equilibrium fails the intuitive criterion.

d. For general A, B, and D, solve for the unique equilibrium which *does* satisfy the intuitive criterion as a function of A, B and D. How does the level of education obtained by the high types vary in D in this equilibrium? What is the intuition?

Problem 3 This problem asks you to consider an extension of the basic Spence model to one in which education is productive and the cost of education is convex.

Suppose that high types with education e have productivity $y(H, e) = 10 + 2e$, while low types have productivity $y(L, e) = 2 + e$. Firms cannot observe whether a worker is a high type or a low type, but know that half of all workers are of each type. A competitive labor market ensures each type of worker is paid her expected productivity. A high type can acquire e units of education at cost $c_H(e) = \frac{1}{10}e^2$, while education costs a low type $c_L(e) = \frac{1}{4}(e + 2)^2 - 1$.

a. Suppose $e_L = 0$ and $e_H = 12$. What wage function would support this outcome as a separating equilibrium? Draw a picture and/or describe using an equation.

b. Does the equilibrium you described in part a satisfy the intuitive criterion? Why or why not?

c. Draw the set of all points which give the high type utility of 5. What is the slope of the indifference curve you drew, as a function of e? Determine the point of tangency between the high type's indifference curve and the function $y(H, e)$ (note that the high type may get more or less than 5 utility at the point of tangency). Do the same for the low type's indifference curve and the function $y(L, e)$.

d. Suppose that both types choose their education level so that their indifference curve is tangent to their productivity function. Describe, using a picture and/or an equation, a wage function that would support this outcome as a perfect Bayesian equilibrium.

e. Does the equilibrium you described in part d satisfy the intuitive criterion? Why or why not?

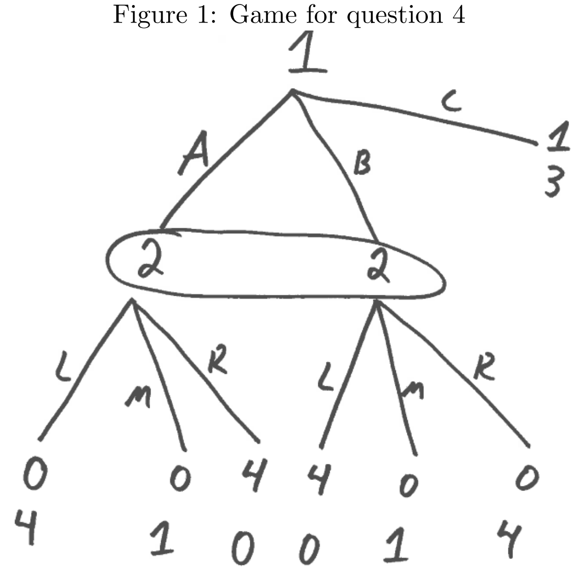
Problem 4 Consider the game in Figure 1 below.

a. Draw the reduced normal form. Find all pure strategy Nash equilibria. There is a mixed Nash equilibrium in which 1 randomizes between A and B, and 2 randomizes between L and R. Find it.

b. Find all of the game's perfect Bayesian equilibria (pure as well as mixed).

c. Explain in intuitive terms any differences between your answers to part a and part b.

Figure 1: Game for question 4



Problem 5 A firm has two types of jobs, good jobs and bad jobs. When a *qualified* worker is assigned to a good job, the firm earns a net profit of \$20,000. When an *unqualified* worker is assigned to a good job, the firm incurs a net loss of \$20,000. When a worker of either type is assigned to a bad job, the firm breaks even. Workers prefer good jobs, and get an extra \$32,000 payoff from a good job relative to a bad job.

To become qualified, a worker pays an investment cost c. This cost is higher for some workers than for others; the distribution of c across all workers is uniform between \$0 and \$9,000. The firm cannot observe which workers are qualified and which are not.

Suppose that while the firm cannot directly observe workers' investment decisions, it administers a test to new employees, with scores ranging from 0 to 1. The probability a qualified worker scores less than $t \in [0, 1]$ is t^2. The probability an unqualified worker scores less than t is t.

a. Suppose that the firm puts all workers with a test score of $s \in [0, 1]$ or higher into a good job. Describe the incentive constraint for a worker's decision to become qualified or not. What fraction π of workers will become qualified, as a function of s?

b. Now consider the firm's problem. Suppose that fraction π of all workers become qualified, so that the firm's prior is π. Show that the firm optimally puts workers scoring above some cutoff test score s into good jobs, and puts low-scoring workers into bad jobs, and solve for s as a function of π.

c. An equilibrium is (s, π) pair such that s maximizes firm profit given π and π is consistent with workers maximizing expected wages net of the investment cost given s. Characterize the equilibrium values of π and s as follows. One, show that $s = \frac{1}{4}$ and $\pi = \frac{2}{3}$ is an equilibrium. Two, show with a picture that there is another equilibrium with $s > \frac{1}{4}$ (you may solve for this equilibrium using a computer if you like, but the answer may not involve round numbers).

d. What economic interpretation does the Coate and Loury paper studied in class assign to the multiplicity of equilibria in its model?

Problem 6 Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$2,000, and if an unqualified worker is assigned to the good job the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than θ is θ^2. The probability that an unqualified worker will have a test score less than θ is θ. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of \$4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and \$3,000, for both A-workers and B-workers.

Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?