Midterm

3/1/2018

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. You have 150 minutes to complete the exam. Good luck!

Problem 1 (20 points) Consider an infinitely-repeated version of the following game with discount factor $\delta < 1$:

	L	C	R
T	6,6	-1,7	-2,8
M	7,-1	4,4	-1,5
B	8,-2	5,-1	0,0

a. For which values of the discount factor δ can the players support the pair of actions (M, C) played in every period? Be sure to describe the strategies you consider.

b. For which values of the discount factor δ can the players support the pair of actions (T, L) played in every period? Why is your answer different from that of part a.?

Problem 2 (15 points) Three firms are considering entering a new market. The total profit obtained by each firm depends on the number of firms that enter. If all three firms enter, each firm loses \$50. If two firms enter, each firm makes \$10. If only one firm enters, that firm makes \$30. Assume entry is costless, and that any firm that does not enter receives a payoff of 0.

a. Find all pure strategy Nash equilibria.

b. Find the symmetric mixed-strategy equilibrium in which all three firms enter with the same probability p. In expectation, what profit does each firm earn in this equilibrium?

Problem 3 (10 points) Draw and/or describe an extensive form game with the property that *decreasing* one of player 2's payoffs *increases* her payoff in the SPE. Describe in plain English what accounts for this seemingly odd result.

Problem 4 (15 points) Consider a common good (e.g., clean air) that is depleted with use. N agents consume the good, and each gets utility both from its own consumption and from the remaining stock of the common good. There is no cost associated with consuming x_i. If agent i consumes x_i, agent i's utility is:

$$u_i(x_i, x_{-i}) = \ln(x_i) + \ln(3000 - \sum_{j=1}^{N} x_j)$$

a. Suppose $N = 2$. Show that the two agents' best response functions are given by:[1]

$$x_1(x_2) = \frac{3000 - x_2}{2}, x_2(x_1) = \frac{3000 - x_1}{2}$$

Solve for the Nash equilibrium values of x_1 and x_2.

b. Now suppose that there are N firms. Solve for the symmetric Nash equilibrium.

[1] Recall that the derivative of $\ln(x)$ is $\frac{1}{x}$.

Problem 5 (20 points) Two duopolists producing differentiated goods face the following demand system:

$$q_1^D = 10 - p_1 + \frac{1}{2}p_2$$
$$q_2^D = 10 - p_2 + \frac{1}{2}p_1$$

Each firm has marginal cost 0. The two firms simultaneously set price and then sell quantity $q_i^D(p_1, p_2)$, $i = 1, 2$.

a. Solve for the Nash equilibrium prices. What profit does each firm earn?

b. Now suppose the firms merge. The demand system remains the same, but now one firm jointly sets both p_1 and p_2. Solve for the merged firm's optimal prices and resulting profit.

Now suppose each firm has a capacity constraint K_i, such that its marginal cost becomes prohibitively high above K_i. For example, K_i may be the capacity of the firm's factory.

c. Suppose $K_1 = K_2 = 6$. Show that in the pre-merger Nash equilibrium, each firm sets a price of \$8. Solve for the prices the merged firm would set following a merger.

d. Suppose $K_1 = K_2 = 4$. Solve for the pre-merger Nash equilibrium prices and the optimal post-merger prices.

e. Are merger price effects affected by binding pre-merger capacity constraints?

Problem 6 (20 points) N identical firms compete *a la Bertrand* by setting price. If one firm sets a lower price than all the others, that firm can sell up to the market demand at that price, $q^D(p)$, while higher-priced firms sell nothing. If more than one firm set the same lowest price, they split the demand evenly. Each firm has a constant marginal cost of c.

a. Suppose that demand is $q^D(p) = 10 - p$ and $c = 2$. What prices will the N firms set in the (symmetric) Nash equilibrium? If the firms were able to form a cartel, with each firm setting the same price, what price would the cartel choose to maximize profits? What profit does each firm each under Nash equilibrium and under the cartel?

The cartel is concerned that its members may cheat, by setting a lower price to steal demand. The cartel proposes to fine any member caught cheating. A fine would permanently lower profits by \$$F$, relative to the collusive profit.[2] You may assume that imposing the fine is a credible threat, but that larger fines are more difficult to implement.

b. Suppose each firm has discount factor $\delta = .8$. Describe the minimum fine $F(N)$ necessary to disincentivize cheating. Is $F(N)$ increasing or decreasing in N?

c. Now suppose each firm has a capacity of $\frac{K}{N}$ (total capacity is split evenly across the firms), with $\frac{q^D(p^c)}{N} < \frac{K}{N} < q^D(p^c)$. A firm cannot sell more than its capacity, regardless of its demand. Describe the minimum fine necessary to disincentivize cheating, $F(N, K)$. Is $F(N, K)$ increasing or decreasing in N?

d. Are more fragmented markets more or less likely to lead to collusion?

[2] That is, if no cheating occurs, all members earn $\pi(p^c)$ in every period, while following being caught cheating, a member would earn $\pi(p^c) - F$ in every future period.