Midterm

answers

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. You have 150 minutes to complete the exam. Good luck!

Problem 1 (20 points) Consider an infinitely-repeated version of the following game with discount factor $\delta < 1$:

	L	C	R
T	6,6	-1,7	-2,8
M	7,-1	4,4	-1,5
B	8,-2	5,-1	0,0

a. For which values of the discount factor δ can the players support the pair of actions (M, C) played in every period? Be sure to describe the strategies you consider. $\delta \geq \frac{1}{5}$.

b. For which values of the discount factor δ can the players support the pair of actions (T, L) played in every period? Why is your answer different from that of part a.? $\delta \geq \frac{1}{4}$. It is easier to support (M, C) than (T, L) because either player can increase his payoff by 25% by deviating from (M, C) and by 33% by deviating from (T, L). Thus, a deviation from (T, L) is relatively more tempting.

Problem 2 (15 points) Three firms are considering entering a new market. The total profit obtained by each firm depends on the number of firms that enter. If all three firms enter, each firm loses \$50. If two firms enter, each firm makes \$10. If only one firm enters, that firm makes \$30. Assume entry is costless, and that any firm that does not enter receives a payoff of 0.

a. Find all pure strategy Nash equilibria. There are three: 1- Firms 1 and 2 enter, firm 3 does not. 2- Firms 1 and 3 enter, firm 2 does not. 3- Firms 2 and 3 enter, firm 1 does not.

b. Find the symmetric mixed-strategy equilibrium in which all three firms enter with the same probability p. In expectation, what profit does each firm earn in this equilibrium? In such an equilibrium, each firm is indifferent between entering and not, meaning that the expected value of entering is 0. If both rivals enter with probability p, then a firm's expected payoff from entering is:

$$\begin{aligned}\pi(enter) =& p^2 * (-50) + 2p(1-p) * 10 + (1-p)^2 * 30 = 0 \\ \iff& 40p^2 + 40p - 30 = 0 \\ \Rightarrow& p = \frac{1}{2}\end{aligned}$$

This indifference condition implies that in the unique symmetric equilibrium, each firm enters with probability $\frac{1}{2}$.

Problem 3 (10 points) Draw and/or describe an extensive form game with the property that *decreasing* one of player 2's payoffs *increases* her payoff in the SPE. Describe in plain English what accounts for this seemingly odd result. Suppose Army 1 can attack or not, and army 2 can defend or retreat a key position overlooking a river. If Army 1 does not attack, both armies receive utility 0. If Army 1 does attack, its payoffs are 10 if Army 2 retreats, and -5 if Army 1 defends, while Army 2's payoffs are -2 from retreating, and -5 from defending. The SPE is for Army 1 to attack and Army 2 to retreat. However, suppose Army 1 were to lower its utility from retreating if attacked to -10 (say, by burning the bridge it would use to retreat, so that it would be an easy target for Army 2). Then, the SPE would shift to Army 1 not attacking and Army 2 defending if attacked. The first SPE gives Army 2 a payoff of -2, while the second gives Army 2 a payoff of 0.

Problem 4 (15 points) Consider a common good (e.g., clean air) that is depleted with use. N agents consume the good, and each gets utility both from its own consumption and from the remaining stock of the common good. There is no cost associated with consuming x_i. If agent i consumes x_i, agent i's utility is:

$$u_i(x_i, x_{-i}) = \ln(x_i) + \ln(3000 - \sum_{j=1}^{N} x_j)$$

a. Suppose $N = 2$. Show that the two agents' best response functions are given by:[1]

$$x_1(x_2) = \frac{3000 - x_2}{2}, x_2(x_1) = \frac{3000 - x_1}{2}$$

Solve for the Nash equilibrium values of x_1 and x_2. Firm 1's maximization problem is:

$$\max_{x_1} \ln(x_1) + \ln(3000 - x_1 - x_2)$$
$$\text{First order condition: } \frac{1}{x_1} = \frac{1}{3000 - x_1 - x_2}$$
$$\Rightarrow x_1 = \frac{3000 - x_2}{2}$$

Agent 2's best response follows from symmetry. Plugging one best response into the other yields a Nash equilibrium of $x_1 = x_2 = 1000$.

b. Now suppose that there are N firms. Solve for the symmetric Nash equilibrium. Firm 1's maximization problem is:

$$\max_{x_1} \ln(x_1) + \ln(3000 - \sum_j x_j)$$
$$\text{First order condition: } \frac{1}{x_1} = \frac{1}{3000 - \sum_j x_j}$$
$$\Rightarrow x_1 = \frac{3000}{N+1} = x_j \text{ (from symmetry)}$$

Problem 5 (20 points) Two duopolists producing differentiated goods face the following demand system:

$$q_1^D = 10 - p_1 + \frac{1}{2}p_2$$
$$q_2^D = 10 - p_2 + \frac{1}{2}p_1$$

Each firm has marginal cost 0. The two firms simultaneously set price and then sell quantity $q_i^D(p_1, p_2)$, $i = 1, 2$.

[1] Recall that the derivative of $\ln(x)$ is $\frac{1}{x}$.

a. Solve for the Nash equilibrium prices. What profit does each firm earn?

The Nash equilibrium is $p_1 = p_2 = \frac{20}{3}$, $\pi_1 = \pi_2 = \frac{400}{9}$.

b. Now suppose the firms merge. The demand system remains the same, but now one firm jointly sets both p_1 and p_2. Solve for the merged firm's optimal prices and resulting profit.

The merged firm sets $p_1 = p_2 = 10$, for a total profit of 100.

Now suppose each firm has a capacity constraint K_i, such that its marginal cost becomes prohibitively high above K_i. For example, K_i may be the capacity of the firm's factory.

c. Suppose $K_1 = K_2 = 6$. Show that in the pre-merger Nash equilibrium, each firm sets a price of \$8. Solve for the prices the merged firm would set following a merger.

Since each firm has a capacity less than the Nash equilibrium quantity of $\frac{20}{3}$, each will increase price until its demand equals its capacity, conditional on its rival's price. Hence, the equations determining price are:

$$6 = 10 - p_1 + \frac{1}{2}p_2$$
$$6 = 10 - p_2 + \frac{1}{2}p_1$$

Jointly solving yields $p_1 = p_2 = 8$. Post-merger, we know from part b. that the merged firm would be unconstrained at its optimal price vector, hence the constraint is irrelevant and it sets $p_1 = p_2 = 10$.

d. Suppose $K_1 = K_2 = 4$. Solve for the pre-merger Nash equilibrium prices and the optimal post-merger prices.

From parts a. and b., here the constraint will bind both before and after the merger, so in both cases prices are determined by:

$$4 = 10 - p_1 + \frac{1}{2}p_2$$
$$4 = 10 - p_2 + \frac{1}{2}p_1$$

Solving, we have that $p_1 = p_2 = 12$ both before and after the merger.

e. Are merger price effects affected by binding pre-merger capacity constraints?

Merger price effects when firms are capacity-constrained depends on how tightly the constraints bind. Post-merger, prices may or may not increase.

Problem 6 (20 points) N identical firms compete *a la Bertrand* by setting price. If one firm sets a lower price than all the others, that firm can sell up to the market demand at that price, $q^D(p)$, while higher-priced firms sell nothing. If more than one firm set the same lowest price, they split the demand evenly. Each firm has a constant marginal cost of c.

a. Suppose that demand is $q^D(p) = 10 - p$ and $c = 2$. What prices will the N firms set in the (symmetric) Nash equilibrium? If the firms were able to form a cartel, with each firm setting the same price, what price would the cartel choose to maximize profits? What profit does each firm each under Nash equilibrium and under the cartel?

It is a standard result that homogeneous Bertrand competitors price at marginal cost, so the Nash equilibrium is $p_i = 2$ for all i. Under a cartel, each firm would set price $p = 6$, and earn a profit of $\frac{16}{N}$.

The cartel is concerned that its members may cheat, by setting a lower price to steal demand. The cartel proposes to fine any member caught cheating. A fine would permanently lower profits by $\$F$, relative to the collusive profit.[2] You may assume that imposing the fine is a credible threat, but that larger fines are more difficult to implement.

b. Suppose each firm has discount factor $\delta = .8$. Describe the minimum fine $F(N)$ necessary to disincentivize cheating. Is $F(N)$ increasing or decreasing in N?

Firm i's incentive constraint is:

$$16 - \frac{16}{N} \leq \frac{\delta}{1-\delta} F$$

Letting this bind with equality, we have that the minimum fine $F(N) = \frac{4(N-1)}{N}$. $F(N)$ is increasing in N, meaning that collusion becomes more difficult as N increases.

c. Now suppose each firm has a capacity of $\frac{K}{N}$ (total capacity is split evenly across the firms), with $\frac{q^D(p^c)}{N} < \frac{K}{N} < q^D(p^c)$. A firm cannot sell more than its capacity, regardless of its demand. Describe the minimum fine necessary to disincentivize cheating, $F(N, K)$. Is $F(N, K)$ increasing or decreasing in N?

Firm i's incentive constraint is:

$$(6-2) * \frac{K}{N} - \frac{16}{N} \leq F$$

Letting this bind with equality, we have that the minimum fine $F(N) = \frac{K-4}{N}$. $F(N)$ is decreasing in N, meaning that collusion becomes easier as N increases.

d. Are more fragmented markets more or less likely to lead to collusion?

The examples in parts c. and d. suggest there is not a straightforward answer to this question.

[2] That is, if no cheating occurs, all members earn $\pi(p^c)$ in every period, while following being caught cheating, a member would earn $\pi(p^c) - F$ in every future period.