Final Exam

answers

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

Problem 1 (20 points) Answer the two questions below. In each case, points will be awarded primarily based on explanations, which should closely relate to concepts from class.

a. The text below excerpts findings by Peter Arcidiaocono, an expert witness for the plaintiffs in litigation against Harvard over their admissions policies.[1] Does the fact pattern described by Prof. Arcidiacono more closely resemble taste-based or statistical discrimination, and why?

> Asian-American applicants to Harvard as a group have, on average, the highest objective academic credentials. In the expanded dataset, their average SAT score (SAT math plus SAT verbal) is 24.9 points higher than white applicants; 153.9 points higher than Hispanic applicants; and 217.7 points higher than African-American applicants. Asian-Americans applicants also have the highest academic index – Harvard's combined score for standardized testing and high-school performance.
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> Despite being more academically qualified than the other three major racial/ethnic groups (whites, African Americans, and Hispanics), Asian-American applicants have the lowest admissions rates. In fact, data produced by Harvard show that this has been true for every admissions cycle for the classes of 2000 to 2019.
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> ...
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> For the Class of 2014 through the Class of 2019, Asian Americans made up roughly 22% of domestic students admitted to the Harvard freshman class. If Harvard relied exclusively on the academic index it assigns to each applicant in making domestic admissions decisions, the Asian-American share of its domestic admitted freshman class over those same six years would be over 50%.

I interpret this as taste-based discrimination. While there are certainly unobservable aspects of college applicants, the Asian-American students appear to be of higher quality than other American students, both on average and at the margin, so it is unlikely that Harvard screens out Asian students because their unobservables are especially poor. As Arcidiacono states, "That the Asian-American admit rate is consistently below the total admit rate over two decades points towards a potential ceiling on the Asian-American admit rate."

b. The text below is from Blair and Chung, 2018.[2] Do the empirical facts they describe more closely resemble taste-based or statistical discrimination, and why?

> The literature shows that ... the black-white gaps in resume callbacks and employment increase in states where employers were restricted from including questions about worker criminal history on job applications (ban-the-box states)... we provide evidence that occupational licensing is an informative job market signal for African-American men. The license serves as a signal of non-felony status, resulting in a higher licensing premium for African-American men in occupations that preclude felons from having a license. In fact, the positive wage benefits of occupational licenses with felony bans are largest for African-American men in ban-the-box states where nonfelony status is harder for employers to deduce.

[1] The full report is available at https://samv91khoyt2i553a2t1s05i-wpengine.netdna-ssl.com/wp-content/uploads/2018/06/Doc-415-1-Arcidiacono-Expert-Report.pdf

[2] See Blair, P. and B. Chung, 2018. "Job market signaling through occupational licensing," *mimeo*.

I interpret this as statistical discrimination. In class, we discussed research by Doleac and Hansen (2017) finding that following "ban the box" legislation preventing employers from conducting criminal background checks until late in the job application process, African-American employment decreased, apparently because (Doleac and Hansen argue) race is strongly correlated with ex-offender status. Blair and Chung find that in "ban the box" states, an occupational license may serve as a signal of non-offender status. Under this interpretation, employers favor those with occupational licenses at least in part because having a license is negatively correlated with ex-offender status.

Problem 2 (20 points) Answer the following two questions about Bayes' rule.

a. The prediction market site predictit.com sells futures contracts that pay \$1 if some future event occurs. The price of the contract, which is set by supply and demand, is equal to the perceived probability that the event will happen. On April 29, 2019, I observed the following prices for contracts paying \$1 if the following individuals win the 2020 democratic nomination for president.

Candidate	price
Bernie Sanders	23 cents
Joe Biden	22 cents
Kamala Harris	16 cents
Pete Buttigieg	15 cents
Elizabeth Warren	8 cents
Beto O'Rourke	6 cents

The candidate who wins the Democratic nomination will run in the general election against Donald Trump in November 2020. The table below shows the price for contracts paying \$1 for each individual winning the November 2020 general election.

Candidate	price
Donald Trump	40 cents
Joe Biden	17 cents
Bernie Sanders	15 cents
Pete Buttigieg	12 cents
Kamala Harris	8 cents
Elizabeth Warren	4 cents
Beto O'Rourke	4 cents

Calculate the probability that each candidate (other than Donald Trump) wins the 2020 general election conditional on winning the democratic nomination. (Hint: a Venn diagram may be particularly helpful in calculating these conditional probabilities). Which democratic candidate has the highest probability of winning the presidency conditional on winning the democratic nomination? Which has the lowest?

Candidate	$P(\text{presidency}\|\text{nomination})$
Bernie Sanders	65.2%
Joe Biden	72.7%
Pete Buttigieg	80%
Kamala Harris	50%
Elizabeth Warren	50%
Beto O'Rourke	66.7%

b. Two players each have a type that is private information. The joint distribution of player types is as follows:

		Player 2's type	
		c	d
Player 1's type	a	$\frac{1}{6}$	$\frac{1}{3}$
	b	$\frac{1}{3}$	$\frac{1}{6}$

Suppose player 1 is type a. Use Bayes' rule to calculate the probability he should assign to player 2 being type c, and to player 2 being type D.

$P(c|a) = \frac{1}{3}$, $P(d|a) = \frac{2}{3}$.

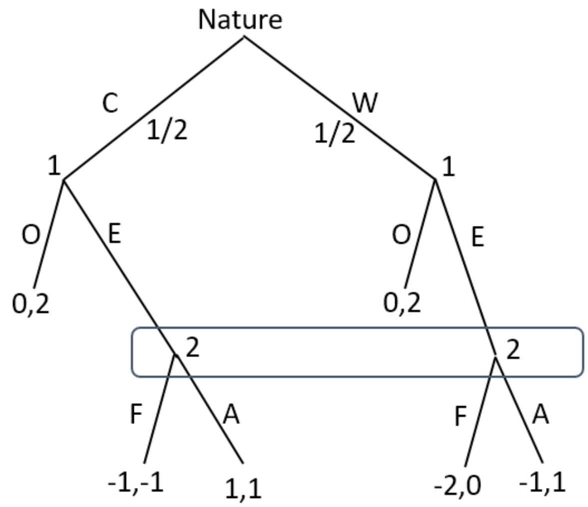

Figure 1: Game for problem 3. Nature is equally likely to play C or W.

Problem 3 (20 points) Consider the modified entry game in figure 1.

a. Find all pure-strategy Nash equilibria of the game (hint: to do this, solve the normal form game in which player 1 has four strategies, OO, OE, EO, and EE, and player 2 has two strategies, F and A. Compute expected payoffs as needed.).

The reduced normal form game has two pure-strategy Nash equilibria, (OO, F) and (EO, A):

Player 2

Player 1		F	A
	OO	$0, 2$	$0, 2$
	OE	$-1, 1$	$-\frac{1}{2}, \frac{3}{2}$
	EO	$-\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{3}{2}$
	EE	$-\frac{3}{2}, -\frac{1}{2}$	$0, 1$

b. Find all pure-strategy perfect Bayesian equilibria of the extensive form game. Don't forget to describe any relevant beliefs. One of the two equilibria solved for in part a. corresponds to a PBE: 1 plays EO, 2 plays A, and 2 believes she is at the left node with 100% probability.

c. Contrast your answers to parts a and b using the language of sequential rationality (e.g., credible threats/promises). The (OO, F) equilibrium is not sequentially rational, because it involves 2 playing a strategy (F) that is suboptimal for any beliefs player 2 might have.

Problem 4 (20 points) Two players participate in a first-price sealed bid auction for a bottle of Scotch. Each player's valuation is $v_i = \frac{1}{2} + \theta_i$, where θ_i is known only to player i, but is drawn from a uniform distribution over $[0, 1]$.

a. Bidder 1 chooses bid b_1 as a function of θ_1 (without knowing θ_2). Describe bidder 1's expected payoff, as a function of b_2 and θ_1. $\max_{b_1} P(b_1 \geq b_2) * (\frac{1}{2} + \theta_1 - b_1)$

b. Suppose bidder 2 sets $b_2 = \frac{1}{2} + \frac{1}{2}\theta_2$. Solve for the value of b_1 that maximizes bidder 1's expected payoff, as a function of θ_1. The following four lines are equivalent:

$$\max_{b_1} P(b_1 \geq b_2) * (\frac{1}{2} + \theta_1 - b_1)$$
$$\max_{b_1} P(\frac{1}{2} + \frac{1}{2}\theta_2 \leq b_1) * (\frac{1}{2} + \theta_1 - b_1)$$
$$\max_{b_1} P(\theta_2 \leq 2b_1 - 1) * (\frac{1}{2} + \theta_1 - b_1)$$
$$\max_{b_1} (2b_1 - 1) * (\frac{1}{2} + \theta_1 - b_1)$$

Thus, bidder 1 has the following first-order condition:

$$2 * (\frac{1}{2} + \theta_1 - b_1) - (2b_1 - 1) = 0$$
$$\Rightarrow 4b_1 = 2 + 2\theta_1$$
$$\Rightarrow b_1 = \frac{1}{2} + \frac{1}{2}\theta_1$$

c. Solve for the Nash equilibrium of this auction game. Given symmetry, and the guess-and-verify answer to part b., each player bidding $b_i = \frac{1}{2} + \frac{1}{2}\theta_i$ is a Nash equilibrium of the game.

Problem 5 (20 points) Firms can assign workers to either *managerial* positions or *non-managerial* jobs. Some workers are *skilled* and some are *unskilled*, but firms cannot observe a worker's skill level directly. When a skilled worker is assigned to a managerial job, the firm earns a net profit of \$20,000. When an unskilled worker is assigned to a managerial job, the firm incurs a net loss of \$20,000. When a worker of either type is assigned to a non-managerial job, the firm breaks even. All workers prefer managerial jobs, and get an extra \$16,000 payoff from a managerial job relative to a non-managerial job.

To become qualified, a worker pays an investment cost c. This cost is higher for some workers than for others; the distribution of c across all workers is uniform between \$0 and \$7,000. The firm cannot observe which workers are qualified and which are not.

Suppose that while the firm cannot directly observe workers' investment decisions, it administers a test to new employees, with scores ranging from 0 to 1. The probability a qualified worker scores less than $t \in [0, 1]$ is $F_q(t) = t^3$ (note that the associated density function is $f_q(t) = 3t^2$). The probability an unqualified worker scores less than t is $F_u(t) = t$ (the associated density function is $f_u(t) = 1$).

Hint: the following equations were used in class:

$$\pi = G\left[\omega(F_u(s) - F_q(s))\right] \quad \text{(workers)}$$
$$\frac{x_q}{x_u} = \frac{1-\pi}{\pi}\frac{f_u(s)}{f_q(s)} \quad \text{(firms)}$$

a. Suppose that the firm puts all workers with a test score of $s \in [0, 1]$ or higher into a management job. Sketch the relationship between s and the fraction of workers that choose to become qualified, given that s. Give an intuitive explanation for the shape of the relationship in your sketch. As seen in class, $\pi(s)$ will be hill-shaped, equal to 0 at $s = 0$ and $s = 1$, and higher in the middle. This is because either a very low or very high cutoff disincentivizes effort, as effort is unlikely to change the outcome.

b. Suppose that the firm believes that fraction π of all workers are qualified. Sketch the firm's optimal choice of s, as a function of π. Give an intuitive explanation for the shape of the relationship in your sketch. As seen in class, $s(\pi)$ is a decreasing function, meaning that the firm sets a lower test score cutoff for workers about whom it has a higher prior.

c. Suppose that the firm believes that $\pi = \frac{4}{7}$ of workers are skilled. What is the minimum test score it will require to place a worker into a managerial job? From the firm's equation, $s(\pi = \frac{4}{7}) = \frac{1}{2}$.

d. Given your answer to part c., what fraction of workers will choose to become skilled? Is a belief that $\frac{4}{7}$ of workers are qualified supportable in an equilibrium? From the workers' equation, $\pi(s = \frac{1}{2}) = \frac{6}{7}$. Thus, such a belief is not supportable as an equilibrium, since more than $\frac{4}{7}$ of workers will choose to become skilled if the firm optimally sets the test score cutoff given this belief.

e. Could this model have more than one equilibrium? You can draw a picture to answer, and you needn't focus on the specific numerical example described here. What economic interpretation does the Coate and Loury paper studied in class assign to the multiplicity of equilibria in its model? Yes. The standard picture discussed in class suffices.