Homework 1

due 2/7/2018

Problem 1 Kirt and Lila are engaged in a joint project. If person $i \in \{K, L\}$ invests effort $x_i \in [0, 1]$ in the project, at cost $c(x_i)$, the outcome of the project is worth $f(x_K, x_L)$. The worth of the project is split equally by Kirt and Lila, regardless of their effort levels, so that each gets a payoff of $\frac{1}{2}f(x_K, x_L) - c(x_i)$. Suppose effort levels are chosen simultaneously.

a. Suppose $f(x_K, x_L) = 3x_K x_L$ and that $c(x_i) = x_i^2$. Find the Nash equilibrium effort levels of this simultaneous move game.

b. Is there a pair of effort levels that yield higher payoffs for both players than do the Nash equilibrium effort levels in part a.?

Problem 2 Consider the normal form game below:

		Avon: I	Avon: N
Joe	I	r, r	$r-1, 0$
	N	$0, r-1$	$0, 0$

In this game, strategy I represents investing, and strategy N represents not investing. Investing yields a payoff of r or $r - 1$, according to whether the player's opponent invests or not. Not investing yields a certain payoff of 0.

Describe the set of Nash equilibria (pure and mixed) of the game for each $r \in [-2, 3]$.

Problem 3 Gibbons, problem 1.13

Problem 4 Three firms are considering entering a new market. The total profit obtained by each firm depends on the number of firms that enter. If all three firms enter, each firm loses \$50. If two firms enter, each firm makes \$10. If only one firm enters, that firm makes \$30. Assume entry is costless, and that any firm that does not enter receives a payoff of 0.

a. Find all pure strategy Nash equilibria.

b. Find the symmetric mixed-strategy equilibrium in which all three firms enter with the same probability p. In expectation, what profit does each firm earn in this equilibrium?

Problem 5 Two duopolists producing differentiated goods face the following demand system:

$$q_1^D = 10 - p_1 + \frac{1}{2}p_2$$
$$q_2^D = 10 - p_2 + \frac{1}{2}p_1$$

Each firm has marginal cost 0. The two firms simultaneously set price and then sell quantity $q_i^D(p_1, p_2)$, $i = 1, 2$.

a. Solve for the Nash equilibrium prices. What profit does each firm earn?

b. Now suppose the firms merge. The demand system remains the same, but now one firm jointly sets both p_1 and p_2. Solve for the merged firm's optimal prices and resulting profit.

Now suppose each firm has a capacity constraint K_i, such that its marginal cost becomes prohibitively high above K_i. For example, K_i may be the capacity of the firm's factory.

c. Suppose $K_1 = K_2 = 6$. Show that in the pre-merger Nash equilibrium, each firm sets a price of \$8. Solve for the prices the merged firm would set following a merger.

d. Suppose $K_1 = K_2 = 4$. Solve for the pre-merger Nash equilibrium prices and the optimal post-merger prices.

e. Are merger price effects affected by binding pre-merger capacity constraints?

Problem 6 99 shepherds share a common field in which they graze their sheep. Each shepherd purchases as many sheep as he/she likes, at a cost of $c = \$300$/sheep. The value of one sheep is given by:

$$v(G) = 2000 - S$$

where S is the total number of sheep which graze in the field (more sheep mean less grass/sheep, more sheep fights, etc). The common field is the only suitable location for grazing, and sheep die without grazing, so you may assume that all purchased sheep are brought to graze in the field.

a. In a symmetric Nash equilibrium, how many sheep does each shepherd purchase? How much profit is earned by each shepherd?

b. What is the socially optimal number of sheep? If the resulting total profit is split evenly amongst all shepherds, what is the profit for each shepherd?

c. Suppose a government imposes a tax on sheep of \$T/head, but that the revenue collected from the tax is distributed evenly to each of the 99 shepherds, regardless of how many sheep the shepherd owns. Is such a tax always welfare-reducing? Why or why not?

Problem 7 Refer to the Games 1 and 2 depicted in Figures 1 and 2.

a. Enter payoffs into the games below so that they have each of the following properties:

- The games are equivalent except for the addition of strategy C to game 2 (i.e. the first two rows of Game 1 are equivalent to the first two rows of Game 2).
- The only Nash equilibrium of Game 1 is (A, L).
- The only Nash equilibrium of Game 2 is (C, R).

b. Referring only to Game 1, enter payoffs such that the only Nash equilibrium is both players mixing with equal probability on each strategy.

Player 2

Player 1		L	R
	A	3,3	
	B		

Figure 1: Game 1

Player 2

Player 1		L	R
	A	3,3	
	B		
	C		2,2

Figure 2: Game 2