Homework 2

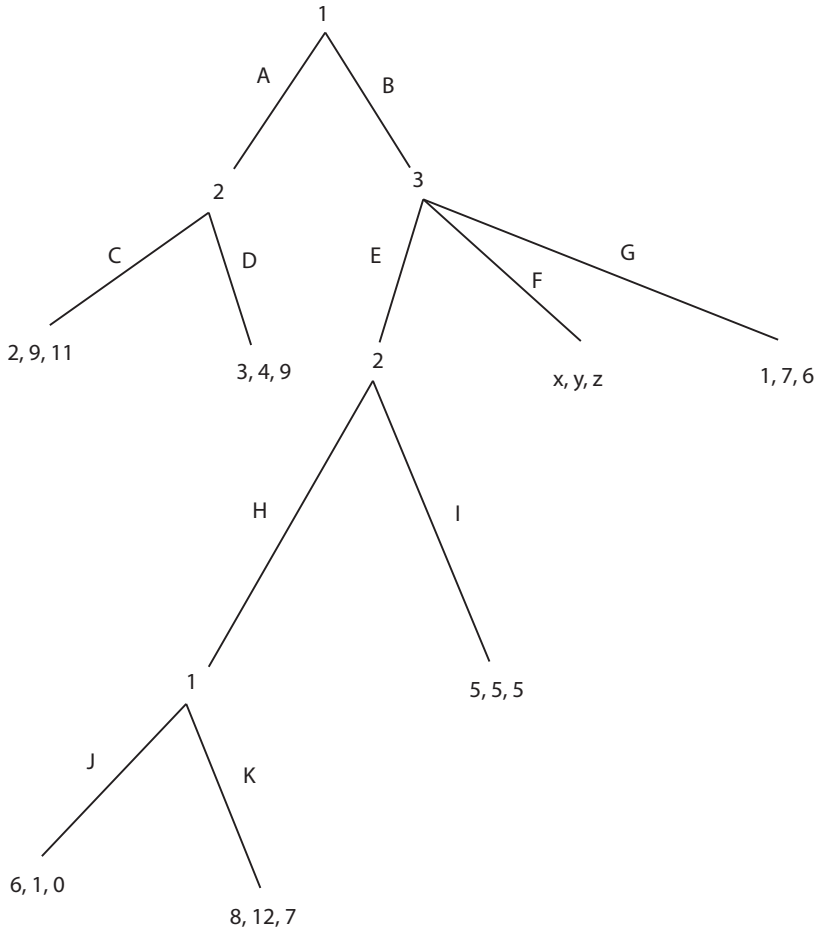
due February 21, 2018

Problem 1 Consider a market with demand curve $P = 1 - Q$, served by Cournot oligopolists with marginal cost c who compete by simultaneously setting quantity.

a. Suppose that there are 2 firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

b. Now suppose that there are N firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

Problem 2 Consider the sequential move game below. Each set of payoffs is ordered u_1, u_2, u_3, where u_i is player i's utility.



For all possible values of x, y, and z, What is the subgame perfect equilibrium of this game? Remember to describe players' choices at all nodes, including those that are unreached.

Problem 3 Consider a market with inverse market demand given by $P = 10 - \frac{1}{100}Q$. Firm A is a monopoly producer, with marginal cost equal to \$2.

a. Calculate Firm A's optimal quantity, and its profit as a monopolist.

Now, suppose that Firm A has discovered a new technology that will allow it to produce at a marginal cost of \$0. Implementing the new technology will cost firm A to incur a fixed cost of \$1,000.

b. Is it profitable for Firm A to implement the new technology?

Now, suppose that Firm A learns that Firm B is considering entering the market to compete with Firm A. To enter, Firm B would have to construct a factory at a cost of \$500, and then Firm A and Firm B would compete in Cournot oligopoly.[1] If firm B entered, its marginal cost would also equal \$2.

c. Calculate the market price, firm A's profit, and firm B's profit under Cournot competition. Would firm B profitably enter the market? (Assume for part c. that Firm A has not implemented the new technology.)

d. Consider an extensive form game with two rounds. In round 1, Firm A decides whether or not to implement the new technology. In round 2, Firm B decides whether or not to enter the market. Using your answers above (and possibly new calculations), determine the subgame perfect equilibrium of this game.

e. Policymakers sometimes worry that monopolists are less likely to innovate than firms in a competitive market.[2] Do your answers above suggest any caveats to this view?

Problem 4 Consider a 2-player bargaining game, in which the players are choosing how to split a surplus of \$1. Each player discounts payoffs one period in the future by $\delta = \frac{1}{2}$. The rules are as follows:

Period 0: Player 1 chooses a value of X between 0 and 1, at cost $c(X) = \frac{3}{8}X^2$.

Period 1: Player 2 makes an offer of $(y, 1-y)$, where y is between 0 and 1. Player 1 may either accept the offer, in which case the surplus is split accordingly and the game ends, or reject the offer, in which case the game moves on to period 2.

Period 2: Player 1 makes an offer to player 2. If the offer is accepted, the surplus is split accordingly and the game ends. If the offer is rejected, a mediator awards a payoff of X to player 1, and $1 - X$ to player 2 (nb. this is the same X chosen by player 1 in period 0; interpret the cost player 1 incurred in that period as that of hiring an attorney to investigate the most favorable mediation venue).

a. Solve for the subgame perfect equilibrium of this game. Is player 1 or player 2 better off in this setup?

b. Intuitively, how would your answer change by an increase in δ? You do not need to work out the mathematical details.

Problem 5 Gibbons, problem 2.7

[1] So that inverse market demand is given by $P = 10 - \frac{1}{100}(q_1 + q_2)$, where q_i is firm i's quantity.

[2] See e.g. "Enhanced market power can also be manifested in... diminished innovation.", page 2 of *Horizonal Merger Guidelines*, 2010, U.S. Department of Justice and Federal Trade Commission.

Problem 6 Alice and Bob compete in a race. At the start of the race, both players are 6 steps away from the finish line. Who gets the first turn is determined by a toss of a fair coin; the players then alternate turns, with the results of all previous turns being observed before the current turn occurs.

During a turn, a player chooses from these four options:

- Do nothing at cost 0;
- Advance 1 step at cost 2;
- Advance 2 steps at cost 7;
- Advance 3 steps of at cost 15.

The race ends when the first player crosses the finish line. The winner of the race receives a payoff of 20, while the loser gets nothing. Assume there is no discounting, but that all else equal each player prefers to finish the game more quickly.

Find the subgame perfect equilibria of this game.[3]

[3]Hint: In the SPE, a player's choice at a decision node only depends on the number of steps he has left and on the number of steps his opponent has left. Make a table with columns and rows numbered from 1-6, representing how many steps each player has left to finish. Solve for what one player will do at each possible state. Since the game is symmetric, solving for what one player will do at each point in your table is sufficient to solve the game.