

Homework 4

due April 4, 2019

Problem 1 A seller has a painting for sale that is either good or bad. A good painting is worth 1 to the seller. A bad painting is worth 0 to the seller. The seller knows the painting's quality. The buyer does not know whether the painting is good or bad, only that it is good with probability $\frac{1}{2}$ and bad with probability $\frac{1}{2}$. A good painting is worth v to the buyer. A bad painting is worth 0 to the buyer.

The buyer makes a one-time offer to the seller, which the seller can accept or reject. To keep the problem simple, assume that the seller accepts offers where she is indifferent.

- a. Suppose $v = 1$. What offer should the buyer make? What is his expected profit?
- b. Suppose $v = 1.5$. What offer should the buyer make? What is his expected profit?
- c. Suppose $v = 5$. What offer should the buyer make? What is his expected profit?
- d. What is the lowest value of v such that both types of the painting are traded in equilibrium?
- e. Discuss the efficiency of the outcome in a., b. and c. What is the source of the inefficiency, if any?

Problem 2 The best available test for Groat's disease is pretty accurate, but sometimes returns false positives and false negatives. Specifically, the test returns a positive result for 92 out of every 100 individuals with Groat's disease (and a negative test result for 8 out of 100 individuals who have Groat's). The test returns a negative result for 99 out of every 100 individuals who do not have Groat's disease (and a positive test result for 1 out of every 100 individuals who do not have Groat's). Only one in ten thousand people has Groat's disease.

- a. Suppose that Alice receives a positive test result. What is the probability that Alice has Groat's? You may assume that prior to taking the test Alice's probability of having Groat's was one in ten thousand.
- b. Suppose a new test is developed which has no false negatives; everyone with Groat's who takes the test receives a positive result. It is still the case that the test returns a negative result for 99 out of 100 people who do not have Groat's. Suppose that Bob takes this new test, and receives a positive result. What is the probability that Bob has Groat's? Again, you may assume that Bob's prior probability of having Groat's is one in ten thousand.
- c. What can you conclude about the effectiveness of preventative screening for rare diseases?

Problem 3 Consider a two-player Bayesian game where both players are not sure whether they are playing game X or game Y, and they both think that the two games are equally likely. This game has a unique Bayesian Nash equilibrium, which involves only pure strategies. What is it? (Hint: start by looking for Player 2's best response to each of Player 1's actions.)

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|----------|---|----------|-----|------|--------|
| | | Player 2 | | | |
| | | L | M | R | |
| Player 1 | T | 1,2 | 1,0 | 1,.3 | Game X |
| | B | 2,2 | 0,0 | 0,3 | |

| | | | | | |
|----------|---|----------|-----|-----|--------|
| | | Player 2 | | | |
| | | L | M | R | |
| Player 1 | T | 1,2 | 1,3 | 1,0 | Game Y |
| | B | 2,2 | 0,3 | 0,0 | |

Problem 4 Now consider a variant of this game (from Problem 2) in which Player 2 knows which game is being played (but Player 1 still does not). This game also has a unique Bayesian Nash equilibrium. What is it? (Hint: Player 2's strategy must specify what she chooses in the case that the game is X and in the case that it is Y.) Compare Player 2's payoff in the games from Problems 2 and 3. What seems strange about this?

Problem 5 Firm 1 is considering taking over Firm 2. It does not know Firm 2's current value, but believes that is equally likely to be any dollar amount from 0 to 100. If Firm 1 takes over firm 2, it will be worth 50% more than its current value, which Firm 2 knows to be x . Firm 1 can bid any amount y to take over Firm 2 and Firm 2 can accept or reject this offer. If 2 accepts 1's offer, 1's payoff is $\frac{3}{2}x - y$, and 2's payoff is y . If 2 rejects 1's offer, 1's payoff is 0 and 2's payoff is x .

- a. Find the unique Bayesian Nash equilibrium of this game.
- b. Can you explain why the result you obtained in part a is sometimes called "adverse selection"? Give two other examples of markets that may exhibit adverse selection.

Problem 6 Two bidders are bidding on a bottle of Scotch in a first-price, sealed bid auction. Bidder 1 values the bottle at v_1 , and bidder 2 values the bottle at v_2 . Neither bidder knows the other's valuation, but each knows that $v_i \sim U[0, 1]$, and that v_1 and v_2 are independent (note that this setting is identical to the first example studied in class). Bidders simultaneously submit hidden bids; the highest bidder gets the bottle for the price he paid.

- a. Show that there is a Nash equilibrium in bidding strategies in which player i bids $b_i = \frac{v_i}{2}$.
- b. Suppose that bidder 2 is irrational, and will bid $b_2 = v_2$. Demonstrate that $b_1 = \frac{v_1}{2}$ remains the best response for player 1.
- c. Suppose a third bidder arrives to bid on the bottle of Scotch. Like bidders 1 and 2, bidder 3's valuation is private information, but is distributed $v_3 \sim U[0, 1]$. v_1 , v_2 , and v_3 are independent. Show that there is a Nash equilibrium in which player i bids $b_i = \frac{2v_i}{3}$.

To answer part c., you will need to use the fact that if X_1 and X_2 are independent $U[0, 1]$ random variables, $P(\max\{X_1, X_2\} < x) = x^2$ for $x \in [0, 1]$.

Problem 7 Consider two Cournot oligopolists, facing demand curve $P = 1 - q_1 - q_2$. Firm 1's marginal cost is as follows:

$$c_1 = 0 \text{ w.p. } (1 - \alpha) \text{ (Firm 1 is low cost)}$$

$$c_1 = X \text{ w.p. } \alpha \text{ (Firm 1 is high cost)}$$

Firm 1 knows its marginal cost, but Firm 2 knows only the distribution given above. Firm 2 has marginal cost equal to 0.

- a. Solve for Firm 1's best response functions. Note that since there are two types of Firm 1 (high and low cost), Firm 1 has two best response functions.
- b. Solve for Firm 2's best response function.
- c. In the oligopoly game's Bayesian Nash equilibrium, what quantity does Firm 2 produce? What quantity does Firm 1 produce if its costs are low? If its costs are high? What is the market price in each case?
- d. What is the derivative of q_1 with respect to X in the case that Firm 1 is a low cost firm? In the case Firm 1 is high cost? Why does Firm 1's quantity depend on X even in the former case?