Homework 4

answers

Problem 1 A seller has a painting for sale that is either good or bad. A good painting is worth 1 to the seller. A bad painting is worth 0 to the seller. The seller knows the painting's quality. The buyer does not know whether the painting is good or bad, only that it is good with probability $\frac{1}{2}$ and bad with probability $\frac{1}{2}$. A good painting is worth v to the buyer. A bad painting is worth 0 to the buyer.

The buyer makes a one-time offer to the seller, which the seller can accept or reject. To keep the problem simple, assume that the seller accepts offers where she is indifferent.

a. Suppose $v = 1$. What offer should the buyer make? What is his expected profit?

For all parts, if the buyer offers any amount in $[0, 1)$, his offer will be rejected, while any offer of at least 1 will be accepted. Given this, the only offers that could possibly be optimal for the buyer are 0 and 1. In part a, if the buyer offers 0, the offer is accepted iff the seller knows the painting to be bad, in which case the painting has no value to the buyer. Should the buyer offer 1, the seller will accept whether the painting is good or bad, meaning that the buyer's expected utility is $\frac{1}{2} * 1 + \frac{1}{2} * 0 - 1 = -\frac{1}{2}$, so the buyer is better off offering 0.

b. Suppose $v = 1.5$. What offer should the buyer make? What is his expected profit?

An offer of 0 nets the buyer utility 0, while an offer of 1 gives the buyer expected utility $\frac{1}{2}*1.5+\frac{1}{2}*0-1 = -.25$. The buyer is better off offering 0 (and thus not getting the painting if it is good).

c. Suppose $v = 5$. What offer should the buyer make? What is his expected profit?

An offer of 0 gives the buyer utility of 0, while an offer of 1 gives $\frac{1}{2} * 5 + \frac{1}{2} * 0 - 1 = 1.5$. Thus, the buyer will offer 1, and will purchase the painting regardless of whether it is a good or bad painting.

d. What is the lowest value of v such that both types of the painting are traded in equilibrium?

$v = 2$.

e. Discuss the efficiency of the outcome in a., b. and c. What is the source of the inefficiency, if any?

The outcomes in part b is inefficient, since regardless of the quality of the painting, it is worth more to the buyer than the seller ($v > 1$), yet the good painting is not traded. In part a, the painting has the same value to either the buyer or the seller, so it is efficient either for the painting to remain with the seller, or for a trade to take place. The outcome in part c is efficient, since the painting is worth more to the seller than the buyer, and both types of painting are traded.

Problem 2 The best available test for Groat's disease is pretty accurate, but sometimes returns false positives and false negatives. Specifically, the test returns a positive result for 92 out of every 100 individuals with Groat's disease (and a negative test result for 8 out of 100 individuals who have Groat's). The test returns a negative result for 99 out of every 100 individuals who do not have Groat's disease (and a positive test result for 1 out of every 100 individuals who do not have Groat's). Only one in ten thousand people has Groat's disease.

a. Suppose that Alice receives a positive test result. What is the probability that Alice has Groat's? You may assume that prior to taking the test Alice's probability of having Groat's was one in ten thousand.

$$\begin{aligned}
P(\text{Groat's}|\text{positive test}) &= \frac{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's})}{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's}) + P(\text{no Groat's}) * P(\text{positive test}|\text{no Groat's})} \\
&= \frac{\frac{1}{10000}\frac{92}{100}}{\frac{1}{10000}\frac{92}{100} + \frac{9999}{10000}\frac{1}{100}} \\
&= .009117
\end{aligned}$$

So, there is less than a 1% chance that someone who tests positive actually has Groat's disease.

b. Suppose a new test is developed which has no false negatives; everyone with Groat's who takes the test receives a positive result. It is still the case that the test returns a negative result for 99 out of 100 people who do not have Groat's. Suppose that Bob takes this new test, and receives a positive result. What is the probability that Bob has Groat's? Again, you may assume that Bob's prior probability of having Groat's is one in ten thousand.

$$\begin{aligned}
P(\text{Groat's}|\text{positive test}) &= \frac{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's})}{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's}) + P(\text{no Groat's}) * P(\text{positive text}|\text{no Groat's})} \\
&= \frac{\frac{1}{10000}}{\frac{1}{10000} + \frac{9999}{10000}\frac{1}{100}} \\
&= .009902 \qquad (1)
\end{aligned}$$

c. What can you conclude about the effectiveness of preventative screening for rare diseases?

For any test of a rare disease with even a slight probability of a false positive, a positive result is much more likely to have come from an individual who does not have the disease than from one who does. Hence, most positive results will be false positives. If false positives are costly, e.g. because of unnecessary surgeries or patient stress, it is not clear that such preventative screening is a good idea.

Problem 3 Consider a two-player Bayesian game where both players are not sure whether they are playing game X or game Y, and they both think that the two games are equally likely. This game has a unique Bayesian Nash equilibrium, which involves only pure strategies. What is it? (Hint: start by looking for Player 2's best response to each of Player 1's actions.)

Game X

Player 1 \ Player 2	L	M	R
T	1,.2	1,0	1,.3
B	2,2	0,0	0,3

Game Y

Player 1 \ Player 2	L	M	R
T	1,.2	1,.3	1,0
B	2,2	0,3	0,0

The unique BNE is (B, L), yielding each player a payoff of 2. Player 1's payoffs do not depend upon which version of the game is actually being played. Her best response to L is to play B and T is a best response to M or R. If 1 plays T, then both M and R give Player 2 an expected utility of .15, so her best response is L. Similarly, Player 2's best response to B is L. So in expected utility, L is a dominant strategy for 2, and 1 best responds with B.

Problem 4 Now consider a variant of this game (from Problem 2) in which Player 2 knows which game is being played (but Player 1 still does not). This game also has a unique Bayesian Nash equilibrium. What is it? (Hint: Player 2's strategy must specify what she chooses in the case that the game is X and in the case that it is Y.) Compare Player 2's payoff in the games from Problems 2 and 3. What seems strange about this?

The unique BNE is (T,(R, M)). Player 2 now knows the game that is being played, and each type of Player 2 has a dominant strategy (R for the type that knows the game is X and M for the type that knows that the game is Y). Since there is no chance that 2 will play L, Player 1's unique best response is to play T. In the first part, each player earned a payoff of 2. In the second part, Player 2 actually has more information about what game is actually being played and ends up only earning 0.3 (in either case). At first it may seem a bit strange that 2 is worse off 1 knowing the game than she is not knowing it. This happens because the uninformed Player 2 uses L as a compromise. When she knows the game, she will choose either M or R, tailoring her action for fit the game. What hurts her is the fact that 1 knows that she knows this information.

Problem 5 Firm 1 is considering taking over Firm 2. It does not know Firm 2's current value, but believes that is equally likely to be any dollar amount from 0 to 100. If Firm 1 takes over firm 2, it will be worth 50% more than its current value, which Firm 2 knows to be x. Firm 1 can bid any amount y to take over Firm 2 and Firm 2 can accept or reject this offer. If 2 accepts 1's offer, 1's payoff is $\frac{3}{2}x - y$, and 2's payoff is y. If 2 rejects 1's offer, 1's payoff is 0 and 2's payoff is x.

a. Find the unique Bayesian Nash equilibrium of this game.

Firm 1 will bid zero and Firm 2 will accept any offer greater than or equal to x. Firm 2 simply accepts offers that are higher than the firm's own value. Firm 1 knows that the value of a firm that accepts an offer of y is anywhere from 0 to y. Thus, the expected value of a firm that accepts is $\frac{y}{2}$, which means that Firm 1's expected payoff as a function of it's bid y is $\frac{3}{2} * \frac{y}{2} = \frac{3}{4}y$. In other words, it expects to lose money on any positive bid it makes. It's best response, then is to bid zero.

b. Can you explain why the result you obtained in part a is sometimes called "adverse selection"? Give two other examples of markets that may exhibit adverse selection.

Adverse selection refers to only "bad" versions of an item being sold for a given price, under uncertain quality. An example would be used car sales. Offering \$4,000 for a certain make, model, and year of a car will only interest sellers who know that their cars are actually worth less than \$4,000. Other examples of markets affected by adverse selection include the dating market[1] and the health insurance market.[2]

Problem 6 Two bidders are bidding on a bottle of Scotch in a first-price, sealed bid auction. Bidder 1 values the bottle at v_1, and bidder 2 values the bottle at v_2. Neither bidder knows the other's valuation, but each knows that $v_i \sim U[0, 1]$, and that v_1 and v_2 are independent (note that this setting is identical to

[1] See "Everything I ever needed to know about economics I learned from online dating," by Paul Oyer (Available on Amazon).

[2] See http://www.nytimes.com/2013/08/04/business/for-obamacare-to-work-everyone-must-be-in.html

the first example studied in class). Bidders simultaneously submit hidden bids; the highest bidder gets the bottle for the price he paid.

a. Show that there is a Nash equilibrium in bidding strategies in which player i bids $b_i = \frac{v_i}{2}$.

Suppose that player 2 bids $b_2 = \frac{v_2}{2}$. We will show that player 1's best response is to bid $b_1 = \frac{v_1}{2}$. By symmetry, then, player 2's best response must also be $b_2 = \frac{v_2}{2}$, and so we will have shown that the proposed bidding strategies comprise a Nash equilibrium.

Player 1 chooses b_1 to maximize his utility:

$$\max_{b_1}(v_1 - b_1) * P(\text{player 1 wins auction}) \tag{2}$$

Given that $b_2 = \frac{v_2}{2}$, and that $v_2 \sim U[0,1]$,

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > \frac{v_2}{2}) \\ &= P(2b_1 > U[0,1]) \\ &= 2b_1 \text{ (so long as } b_1 \leq \tfrac{1}{2}\text{. The probability is 1 for } b_1 > \tfrac{1}{2}.) \end{aligned}$$

Hence, initial becomes:

$$\max_{b_1}(v_1 - b_1) * 2b_1$$

which has FOC $2v_1 = 4b_1$, or $b_1 = \frac{v_1}{2}$, as was to be shown.

b. Suppose that bidder 2 is irrational, and will bid $b_2 = v_2$. Demonstrate that $b_1 = \frac{v_1}{2}$ remains the best response for player 1.

Going back to initial, we have that:

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > v_2) \\ &= P(b_1 > U[0,1]) \\ &= b_1 \end{aligned}$$

Hence, initial becomes:

$$\max_{b_1}(v_1 - b_1) * b_1$$

which has FOC $b_1 = \frac{v_1}{2}$, as was to be shown.

c. Suppose a third bidder arrives to bid on the bottle of Scotch. Like bidders 1 and 2, bidder 3's valuation is private information, but is distributed $v_3 \sim U[0,1]$. v_1, v_2, and v_3 are independent. Show that there is a Nash equilibrium in which player i bids $b_i = \frac{2v_1}{3}$.

To answer part c., you will need to use the fact that if X_1 and X_2 are independent $U[0,1]$ random variables, $P(\max\{X_1, X_2\} < x) = x^2$ for $x \in [0,1]$.

Going back to $\mathring{\text{i}}$nitial, we have that:

$$\begin{aligned} P(\text{player 1 wins auction}) &= P(b_1 > max\{v_2, v_3\}) \\ &= b_1^2 \end{aligned}$$

Hence, player 1's FOC is $3b_1^2 - 2b_1v_1 = 0$, or $b_1 = \frac{2}{3}v_1$, as was to be shown.

Problem 7 Consider two Cournot oligopolists, facing demand curve $P = 1 - q_1 - q_2$. Firm 1's marginal cost is as follows:

$$c_1 = 0 \text{ w.p. } (1-\alpha) \text{ (Firm 1 is low cost)}$$
$$c_1 = X \text{ w.p. } \alpha \text{ (Firm 1 is high cost)}$$

Firm 1 knows its marginal cost, but Firm 2 knows only the distribution given above. Firm 2 has marginal cost equal to 0.

a. Solve for Firm 1's best response functions. Note that since there are two types of Firm 1 (high and low cost), Firm 1 has two best response functions.

Low cost Firm 1: $q_1^L = \frac{1}{2} - \frac{1}{2}q_2$. High cost Firm 1: $q_1^H = \frac{1}{2} - \frac{1}{2}q_2 - \frac{1}{2}X$.

b. Solve for Firm 2's best response function.

$q_2 = \frac{1}{2} - (1-\alpha)\frac{1}{2}q_1^L - \alpha\frac{1}{2}q_1^H$.

c. In the oligopoly game's Bayesian Nash equilibrium, what quantity does Firm 2 produce? What quantity does Firm 1 produce if its costs are low? If its costs are high? What is the market price in each case?

$$q_1^L = \frac{1}{3} - \frac{1}{6}\alpha X$$
$$q_1^H = \frac{1}{3} - \frac{1}{6}\alpha X - \frac{1}{2}X$$
$$q_2 = \frac{1}{3} + \frac{1}{3}\alpha X$$

If Firm 1's costs are high, the price is $P^H = \frac{1}{3} - \frac{1}{6}\alpha X + \frac{1}{2}\alpha X$. if Firm 1's costs are low, the price is $P^L = \frac{1}{3} - \frac{1}{6}\alpha X$.

d. What is the derivative of q_1 with respect to X in the case that Firm 1 is a low cost firm? In the case Firm 1 is high cost? Why does Firm 1's quantity depend on X even in the former case?

$\frac{\partial q_1^L}{\partial X} = -\frac{1}{2}\alpha$. $\frac{\partial q_1^H}{\partial X} = -\frac{1}{2}\alpha - \frac{1}{2}$. Both are negative. Even when Firm 1 is low-cost, a higher value of X causes Firm 2 to produce more, which causes Firm 1 to produce less.