Homework 5

due April 18, 2019

Problem 1 Suppose that *normal* workers have productivity of \$6, while *smart* workers have productivity of \$A, where $A > 6$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's education level e. Firms know that half of all workers are normal, and half are smart.

Any worker can acquire as much education as she wishes, but getting e units of education costs a normal worker $B * e$, where $B > 1$, and costs a smart worker e. Assume the labor market is competitive, so that a worker earns her expected productivity. A worker's lifetime utility function is her wage minus the cost of any education she receives.

a. Suppose $A = 20$ and $B = 2$. In a graph with e on the X-axis, and wage on the Y-axis, draw 3 indifference curves for both smart and normal workers. You have enough information for your drawing to be precise.

b. Suppose $A = 20$ and $B = 2$. Construct a wage function so that there is a pooling equilibrium, with both smart and normal workers obtaining 3 units of education. Describe the wage function you chose using a graph (and, if possible, an equation).

c. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part b does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

d. Suppose that $A = 20$ and $B = 2$. Construct a wage function so that there a separating equilibrium in which normal types get education $e_N = 0$, while smart types gets $e_S = 10$. Depict the equilibrium graphically.

e. Use a new graph and a verbal explanation to demonstrate that the equilibrium you constructed in part d does not satisfy the intuitive criterion. Clearly state which part of your wage function fails the criterion.

f. Describe, using a graph and words, the unique equilibrium outcome (e_N, e_S) of this game that satisfies the intuitive criterion.

g. For general values of A and B, determine the unique equilibrium outcome (e_N, e_S) satisfying the intuitive criterion.

h. Explain verbally how the outcome in g is affected by an increase in A. Explain intuitively why this is the case. Do the same for an increase in B.

Problem 2 This problem asks you to consider an extension of the basic Spence model to one in which education is productive and the cost of education is convex.

Suppose that high types with education e have productivity $y(H, e) = 10 + 2e$, while low types have productivity $y(L, e) = 2 + e$. Firms cannot observe whether a worker is a high type or a low type, but know that half of all workers are of each type. A competitive labor market ensures each type of worker is paid her expected productivity. A high type can acquire e units of education at cost $c_H(e) = \frac{1}{10}e^2$, while education costs a low type $c_L(e) = \frac{1}{4}(e + 2)^2 - 1$.

a. Suppose $e_L = 0$ and $e_H = 12$. What wage function would support this outcome as a separating equilibrium? Draw a picture and/or describe using an equation.

b. Does the equilibrium you described in part a satisfy the intuitive criterion? Why or why not?

c. Draw the set of all points which give the high type utility of 5. What is the slope of the indifference curve you drew, as a function of e? Determine the point of tangency between the high type's indifference curve and the function $y(H, e)$ (note that the high type may get more or less than 5 utility at the point of tangency). Do the same for the low type's indifference curve and the function $y(L, e)$.

d. Suppose that both types choose their education level so that their indifference curve is tangent to their productivity function. Describe, using a picture and/or an equation, a wage function that would support this outcome as a perfect Bayesian equilibrium.

e. Does the equilibrium you described in part d satisfy the intuitive criterion? Why or why not?

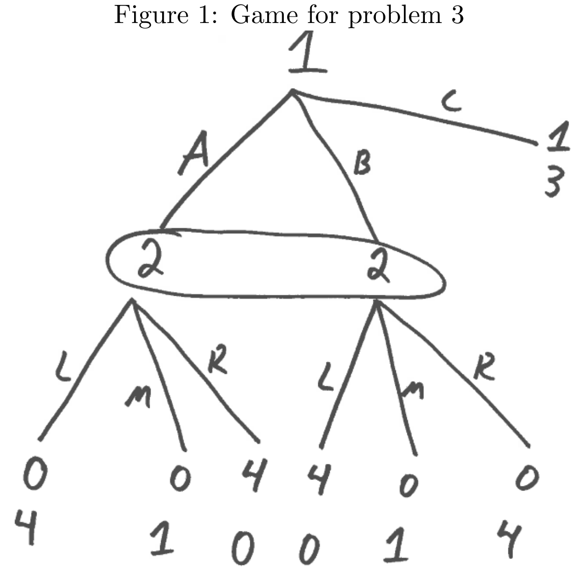
Problem 3 Consider the game in Figure 1 below.

a. Draw the reduced normal form. Find all pure strategy Nash equilibria. There is a mixed Nash equilibrium in which 1 randomizes between A and B, and 2 randomizes between L and R. Find it.

b. Find all of the game's perfect Bayesian equilibria (pure as well as mixed).

c. Explain in intuitive terms any differences between your answers to part a and part b.

Figure 1: Game for problem 3



Problem 4 Consider a signaling game that satisfies:

1. Two types of player 1, tough and weak. Player 1's type is unobservable to player 2, but known to player 1. Each type is equally likely *ex ante*, and is chosen by nature.

2. Player 1 encounters player 2 in a competition for resources.

3. Either type of player 1 sends one of two signals. He can flee (game ends, player 1 receives 0, player 2 receives 2), or he can engage in costly aggressive behavior.

4. If player 1 behaves aggressively, player 2 can fight or flee. In this case, payoffs are as follows (the first number is player 1's payoff):

	fight	flee
tough	$-2, -2$	$1, 0$
weak	$-4, 4$	$-1, 0$

a. Draw the extensive form of the game described above (hint: your picture will be similar, though not identical, to the signaling games we studied in class).

b. Is there a pooling equilibrium in which both types of player 1 behave aggressively? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion.

c. Is there a pooling equilibrium in which both types of player 1 flee? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion.

d. Is there a separating equilibrium in which the tough type behaves aggressively and the weak type flees? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion.

Problem 5 Consider a version of the Cournot oligopoly game in which firm 2's costs are unknown to firm 1. Firm 2 knows its own cost, however. Specifically,

Inverse demand: $P = 1 - q_1 - q_2$

Firm 1's marginal cost: 0

$$\text{Firm 2's marginal cost: } \begin{cases} c_L = .2 & \text{with probability } \frac{1}{4} \\ c_H = .4 & \text{with probability } \frac{3}{4} \end{cases}$$

a. Suppose firm 2 is low cost (so that its marginal cost is $c_L = .2$). Solve for the value of q_2^L that maximizes firm 2's profits. (hint: your answer should be a function of q_1)

b. Suppose firm 2 is high cost (so that its marginal cost is $c_L = .4$). Solve for the value of q_2^H that maximizes firm 2's profits.

c. Solve for the value of q_1 that maximizes firm 1's profits for any values of q_2^L and q_2^H.

d. Clearly describe the oligopoly game's Bayesian Nash equilibrium.