Midterm

March 7, 2019

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. You have 150 minutes to complete the exam. Good luck!

Questions 1-2 pertain to the game in Figure 1:

		Player 2	
		L	R
Player 1	U	10,10	4,50
	D	50,4	0,0

Figure 1: Problems 1 and 2 of this exam refer to this normal form game.

Problem 1 (20 points) Consider the game in Figure 1:

a. Find all Nash equilibrium strategies, pure as well as mixed.

b. For each of the Nash equilibria you solved for in a, state the payoff received by each player.

c. Solve for each player's minmax payoff. Also state the strategy each player uses to minmax his/her opponent.

d. Now suppose the game is played sequentially, with player 1 choosing his strategy first, and player 2 observing 1's choice prior to choosing her strategy. Solve for all subgame perfect equilibria of this game.

Problem 2 (20 points) Now, suppose the game in Figure 1 is played repeatedly ad infinitum, with both players sharing a discount factor $\delta \in (0, 1)$.

a. What is the minimum value of δ for which (U, L) is sustainable in a SPE using grim trigger Nash reversion strategies?

Now, consider a carrot and stick strategy, in which (U, L) is played initially, and players switch to (D, R) for T periods following any deviation from (U, L). If anything other than (D, R) is played during the punishment phase, the punishment restarts, meaning that (D, R) is played for an additional T periods. After T periods of (D, R), the game restarts at (U, L).

b. What condition on δ and T must hold for neither player to wish to deviate from (U, L) in phase 1 of the game?

c. What condition on δ and T must hold for neither player to wish to deviate from (D, R) in phase 2 of the game?

d. Suppose $\delta = .9$. Go as far as you can in determining for which values for T do the carrot and stick strategies comprise a subgame perfect equilibrium.

e. Describe in intuitive terms, why carrot and stick strategies fail SPE equilibrium conditions if T is very small or very large.

Problem 3 (20 points) Two players must divide a surplus of one million dollars. Suppose they play the following bargaining game:

Period 1: Player 1 makes an offer to player 2 on how to divide the million dollars. If 2 accepts, the game ends. If 2 rejects the offer, the game moves to period 2.

Period 2: Player 2 makes an offer to player 1 on how to divide the million dollars. If 1 accepts, the game ends. If 1 rejects the offer, the game moves to period 3.

Period 3: Player 1 makes an offer to player 2 on how to divide the million dollars. If 2 accepts, the game ends. If 2 rejects the offer, both players immediately receive \$100,000, while the remaining \$800,000 is lost (e.g., to lawyers).

Players discount payoffs one period in the future by δ.

a. Solve for the game's subgame perfect equilibrium. Make sure to list what offer will be made in each period, in what period (if any) an offer will be accepted, and how the surplus is divided.

b. Is player 2's equilibrium payoff (measured in period 1) increasing, decreasing, or nonmonotonic in the discount factor δ. Why is this?

For parts c. and d., suppose that player 2 has the option to pay a bribe of $\$X$ at the beginning of period 1. Paying the bribe would increase the amount she would receive should the offer in period 3 be rejected to \$200,000. No other aspect of the game is affected by the bribe.

c. Suppose the discount factor is $\delta = .9$. What is the maximum bribe she would be willing to pay?

d. If the discount factor were *smaller* than .9, would the maximum bribe player 2 would be willing to play increase or decrease? Why?

Problem 4 (20 points) Firm A is a monopoly seller of an intermediate good (e.g., glass for phone screens). For simplicity, suppose A's marginal cost of production is 0.

A sells its output to firm B for price c. Suppose firm B uses exactly one unit of the intermediate good to produce one unit of its final good (e.g., phones), and (for simplicity) that B has no costs other than the cost of the intermediate good purchased from A. Demand for B's final good is given by $Q = 1 - P$.

Suppose firms A and B interact as follows. First, A chooses a price for the intermediate good, c. Then, B observes c and chooses a price for its final good. Both A and B wish to maximize their profits.

a. Determine B's profit-maximizing price, as a function of c (hint: B takes c as fixed when choosing price).

b. Determine the quantity of the intermediate good B purchases, as a function of c (hint: recall that B needs exactly one unit of the intermediate good for each copy of the final good it sells).

c. Determine the value of c that maximizes firm A's profit. What quantity of the final good is produced?

Now, suppose that firm A and B merge. The merged firm maximizes the sum of A's profit and B's profit.

d. Solve for the profit maximizing values of c and P. What quantity of the final good is produced?

e. Combinations of producers of intermediate and final goods are often referred to as *vertical mergers.* Based on your answers to parts c. and d., do vertical mergers tend to increase or decrease prices of final goods?

Problem 5 (20 points) Consider a common good (e.g., clean air) that is depleted with use. N agents consume the good, and each gets utility both from its own consumption and from the remaining stock of the common good. There is no cost associated with consuming x_i. If agent i consumes x_i, agent i's utility is:

$$u_i(x_i, x_{-i}) = \ln(x_i) + \ln(6000 - 2\sum_{j=1}^{N} x_j)$$

a. Suppose $N = 2$. Show that the two agents' best response functions are given by:[1]

$$x_1(x_2) = 1500 - \frac{1}{2}x_2, \quad x_2(x_1) = 1500 - \frac{1}{2}x_1$$

Solve for the Nash equilibrium values of x_1 and x_2.

b. Now suppose that there are N agents. Solve for the symmetric Nash equilibrium, in which all agents choose the same value of x.

[1] Recall that the derivative of $\ln(x)$ is $\frac{1}{x}$.