Homework 1

answers

Disclaimer: I will be quite brief in writing out answer keys to HWs and exams in this class. Sometimes I will merely list the correct final answer. When you are completing HW's, and especially when you are answering exam and prelim questions, you should take the opposite approach: be as verbose as time permits. Do not list calculations without explaining what they mean and why you are doing them, in words. Do not draw pictures unless you have text accompanying them, explaining the picture. A well-reasoned, thoughtful answer that arrives at a grievously wrong conclusion will be graded more generously than a 100% correct answer that is explained poorly. You should view HW's as an opportunity to practice the skill of explaining your answers.

Problem 1 Suppose a player in an extensive form game has m information sets and that at his k^{th} information set, he can choose from among b_k actions.

i. How many pure strategies does this player have? $\prod_{k=1}^{m} b_k$

ii. What is the dimension of his set of mixed strategies? What is the dimension of his set of behavior strategies? (Hint: "dimension" means how many pieces of information you would need to completely understand his mixed or behavior strategy.)His mixed strategy set has dimension $(\prod_{k=1}^{m} b_k) - 1$ (as probabilities always sum to 1, the probability assigned to the final strategy is implied by knowing every other probability). His set of behavior strategies has dimension $\sum_{k=1}^{m}(b_k - 1) = (\sum_{k=1}^{m} b_k) - m$.

Problem 2 Give an example of a game in which a player has a mixed strategy which does not admit an equivalent behavior strategy. Suppose 1 initially chooses U or D. If he chooses U, 2 chooses from l or r. If D, 2 chooses p or q. l or q ends the game, r or p both lead to the same information set for 1, at which he chooses A or B, and the game ends. Consider the mixed strategy σ_1, given by $\sigma_1 = (\sigma_1(UA) = 0, \sigma_1(UB) = \frac{1}{3}, \sigma_1(DA) = \frac{1}{3}, \sigma_1(DB) = \frac{1}{3})$. Clearly, any equivalent behavior strategy must have both $b(U) > 0$ and $b(A) > 0$, yet this would imply $\sigma_1(UA) > 0$, a contradiction. Note that this game does not exhibit perfect recall.

Problem 3 In the game Γ, player 1 moves first, choosing between actions A and B. If he chooses B, then player 2 chooses between actions C and D. If she chooses D, then player 1 moves again, choosing between actions E, F, and G. A choice of A or C ends the game. Payoffs are irrelevant for this question.

i. Find a behavior strategy which is equivalent to the following mixed strategy:

$$\sigma_1 = (\sigma_1(AE).\sigma_1(AF), \sigma_1(AG), \sigma_1(BE), \sigma_1(BF), \sigma_1(BG)) = (\frac{1}{2}, \frac{1}{3}, 0, 0, \frac{1}{12}, \frac{1}{12})$$

$b_1(A) = \sigma_1(AE) + \sigma_1(AF) + \sigma_1(AG) = \frac{5}{6}$. $b_1(E) = \frac{\sigma_1(BE)}{\sigma_1(BE)+\sigma_1(BF)+\sigma_1(BG)} = 0$. $b_1(F) = \frac{1}{2}$.

ii. Describe *all* mixed strategies which are equivalent to the following behavior strategy:

$$b_1 = ((b_1(A), b_1(B)), (b_1(E), b_1(F), b_1(G))) = ((\frac{1}{3}, \frac{2}{3}), (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}))$$

Any σ_1 satisfying the following is equivalent to the given behavior strategy: $\sigma_1(AE)+\sigma_1(AF)+\sigma_1(AG) = \frac{1}{3}$ $\sigma_1(BF) = \frac{1}{6}$, $\sigma_1(BG) = \frac{1}{6}$, $\sigma_1(BE) = \frac{1}{3}$.

Problem 4 In the game Γ', player 1 moves first, choosing between actions L and R. Player 2 observes this choice. If 1 chooses L, then 2 chooses between actions A and B. If 1 chooses R, then 2 chooses between actions C and D. L * Payroll Deductioet $b_2 = ((b_2(A), b_2(B)), (b_2(C), b_2(D))$ be a behavior strategy for player 2.

i. Describe the collection of mixed strategies which are equivalent to b_2. (Hint: you will need to write down equations describing the relationship between b_2 and $\sigma_2(AC), \sigma_2(AD)$, and so on.) Any σ_2 satisfying the following four equations is equivalent to the given behavior strategy:

$$\sigma_2(AC) + \sigma_2(AD) = b_2(A) \tag{1}$$

$$\sigma_2(BC) + \sigma_2(BD) = b_2(B) \tag{2}$$

$$\sigma_2(AC) + \sigma_2(BC) = b_2(C) \tag{3}$$

$$\sigma_2(AD) + \sigma_2(BD) = b_2(D) \tag{4}$$

ii. Specify a single mixed strategy which is equivalent to b_2. (Hint: were you to have numbers for b_2, your answer should describe how to use those numbers to get a specific mixed strategy σ_2. There are many correct answers.) The most straightforward answer is $\sigma_2(AC) = b_2(A)b_2(C)$, $\sigma_2(AC) = b_2(A)b_2(D)$, etc.

Problem 5 Dave has preferences over lotteries which assign probabilities $p = (p_a, p_b, p_c)$ to three possible prizes: an apple, a banana, and a cherry. Suppose that Dave is indifferent between the lottery $p^1 = (1, 0, 0)$ and $p^2 = (0, \frac{1}{2}, \frac{1}{2})$, and that Dave strictly prefers the lottery $p^3 = (\frac{1}{2}, \frac{1}{2}, 0)$ to the lottery $p^4 = (0, \frac{3}{4}, \frac{1}{4})$. Are these preferences consistent with the von Neumann-Morgenstern axioms? (hint: when you are asked questions like this on HWs/exams, the answer is usually no, as it is much easier to disprove a statement like this than it is to prove it). Dave's preferences fail to satisfy the independence axiom. Let $q = (0, 1, 0)$. Then, $p^3 = \frac{1}{2}p^1 + \frac{1}{2}q$, and $p^4 = \frac{1}{2}p^2 + \frac{1}{2}q$. Hence, the independence axiom implies that Dave's preference between between p^3 and p^4 must be the same as his preference between p^1 and p^2, but the question states that this is not the case. Alternatively, one could show that if the utilities u_a, u_b and u_c satisfy $u_a = \frac{1}{2}u_b + \frac{1}{2}u_c$, they must also satisfy $\frac{1}{2}u_a + \frac{1}{2}u_b = \frac{3}{4}u_b + \frac{1}{4}u_c$, which is impossible. As Dave's preferences do not admit an expected utility representation, they cannot satisfy all three axioms.

Problem 6 Find the reduced normal forms of the games in Figures 1 and 2. (Hint: for the 3 player game in figure 1, you will need to draw two payoff matrices, one for each of player 3's actions.) For Figure 1:

2

		a	d
1	A	2, 2, 2	0, 0, 1
	D	0, 0, 3	0, 0, 3

3 plays L

2

		a	d
1	A	2, 2, 2	0, 3, 3
	D	1, 0, 2	1, 0, 2

3 plays R

For figure 2:

2

		Ll	Lr	Rl	Rr
1	AD	4, 3	4, 3	4, 4	4, 4
	AE	3, 9	3, 9	4, 4	4, 4
	B	3, 3	3, 3	8, 2	8, 2
	C	0, 2	6, 1	0, 2	6, 1

Note that I have combined the redundant strategies BD and BE, and CD and CE for player 1.

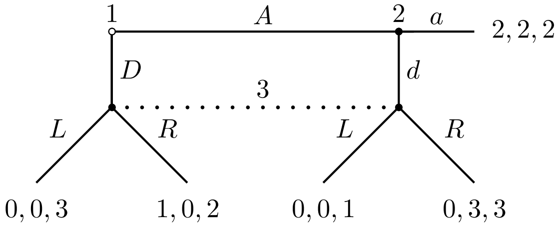

Figure 1: *Seltens horse*

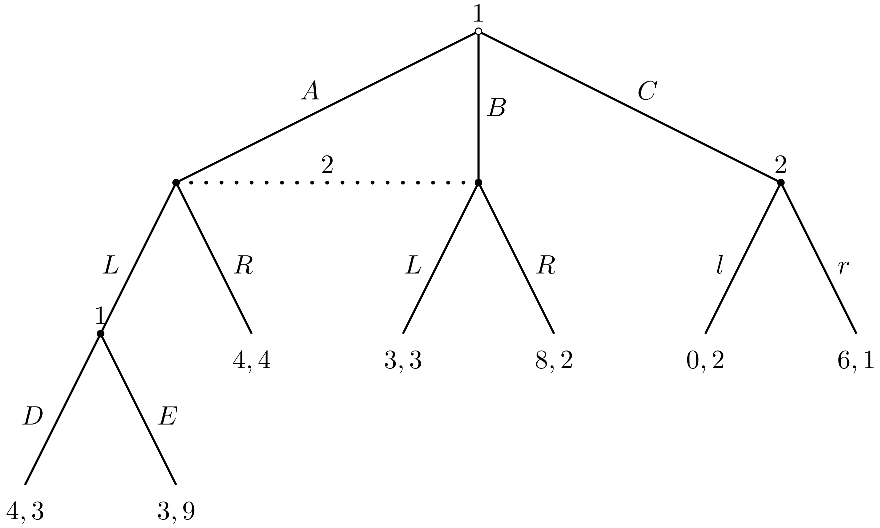

Figure 2: The unblinking eye