Homework 5

answers

Problem 1 Suppose market demand is given by $p(q) = a - bq$, and there are two firms, each with a constant marginal costs of c and no fixed cost. The two firms choose quantity simultaneously, and then sell whatever they have produced at the prevailing market price.

a. Determine NE quantities for both firms. Demonstrate that there is only one equilibrium in this game.

Firm i's best response function is $q_i = \frac{a-c}{2b} - \frac{q_{-i}}{2}$, for $i = 1, 2$. Clearly, this is linearly decreasing in q_{-i}, and so there can be at most one Nash equilibrium (draw a picture to convince yourself of this). The unique Nash equilibrium is located at $(q_1, BR_2(q_1))$ satisfying $q_1 = BR_1(BR_2(q_1))$, or $q_1 = q_2 = \frac{a-c}{3b}$.

b. Derive the market price, and the profit for each firm. Show that the total quantity produced is greater than the monopoly quantity, but less than the competitive quantity.

The market price is $\frac{a+2c}{3}$, while each firm earns a profit of $\pi^c = \frac{(a-c)^2}{9b}$.

c. How high would δ need to be for there to be a SPE of the repeated game in which firm 1 receives fraction α of the monopoly profit and firm 2 receives fraction $1 - \alpha$? Make sure to say how your answer depends on α, including pointing out for what ranges of α no such equilibrium is possible.

First of all, this question is not answerable without using Nash reversion strategies, so let's confine attention to those. The monopoly profit is $\pi^m = \frac{(1-c)^2}{4b}$, and the monopoly quantity is $q^m = \frac{a-c}{2b}$. Consider repeated game strategies where, wlog, player 1 plays αq^m on the equilibrium path, player 2 plays $(1-\alpha)q^m$ on the equilibrium path, and any deviation is met with permanent Nash reversion, i.e. each player earning the Cournot profit of $\frac{(a-c)^2}{9b}$. It should be clear that while either player has a short-run incentive to deviate from the equilibrium path, player 1 will be more likely to do so, and so if the common discount factor is high enough so that player 1 does not want to deviate, player 2 will not want to deviate either.

Now, 1's profit-maximizing short-run deviation is $\frac{a-c}{2b} - (1-\alpha)\frac{a-c}{4b} = \frac{a-c}{4b}(1+\alpha)$, which will induce a price of $a - (1+\alpha)\frac{a-c}{4} - (1-\alpha)\frac{a-c}{2}$, and give firm 1 a one-period profit of $\pi^{dev} = \frac{a-c}{4b}(1+\alpha)\left(a - (1+\alpha)\frac{a-c}{4} - (1-\alpha)\frac{a-c}{2}\right)$.

Therefore, player 1 prefers not to deviate from the equilibrium path iff

$$\begin{aligned}
\alpha\pi^m &\geq (1-\delta)\pi^{dev} + \delta\pi^c \\
\alpha\pi^m &\geq \pi^{dev} + \delta(\pi^c - \pi^{dev} \\
\delta(\pi^{dev} - \pi^c) &\geq \pi^{dev} - \alpha\pi^m \\
\delta &\geq \frac{\pi^{dev} - \alpha\pi^m}{\pi^{dev} - \pi^c} \qquad (1)
\end{aligned}$$

It's quite possible that (1) would simplify usefully if needed. Note that for such a strategy to work at all, we need $\alpha\pi^m \geq \pi^c$, or $\alpha \geq \frac{\pi^c}{\pi^m}$. It is also clear from four that if δ is high enough for player 1 to not want to deviate from the equilibrium path, it is also high enough so that player 2 does not want to deviate. Since we are using Nash reversion, clearly no one wants to deviate from the punishment path.

d. Now suppose the game is played only once, but in which firm 1 moves first. Firm 2 moves only after observing the quantity firm 1 chooses. Derive the SPE of this game.

Now firm 1's maximization problem changes to the following:

$$\max_{q_1}(a - bq_1 - b(\frac{a-c}{2b} - \frac{q_1}{2}))q_1 - cq_1$$
$$= \max_{q_1}(\frac{a-c}{2} - b\frac{q_1}{2})q_1$$

which has FOC $q_1^s = \frac{a-c}{2b}$. Plugging this into 2's best response function gives $q_2^s = \frac{a-c}{4b}$. The market price is $p^s = \frac{1}{4}a + \frac{3}{4}c$, firm 1's profit is $\pi_1^s = \frac{(a-c)^2}{8b}$, and firm 2's profit is $\pi_2^s = \frac{(a-c)^2}{16b}$.

e. Finally, suppose there are J firms serving the market. In the static case, determine NE quantities and profits for each of the J firms. Show that as $J \to \infty$, total production approaches the competitive level, while when $J = 1$, we get the monopoly outcome.

The Nash equilibrium is for each firm to produce $q = \frac{a-c}{b(J+1)}$, leading to a market price of $a - (a-c)\frac{J}{J+1}$ and per firm profit of $\frac{(a-c)^2)}{(J+1)^2 b}$. Clearly, as $J \to \infty$, price converges to c, per-firm profits converge to zero, and total quantity supplied converges to $\frac{a-c}{b}$, the efficient quantity.

Problem 2 The Phoenix Moons, a pro football team, have a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is \$30, yet this results in an average attendance of only 20,000 people.

a. Explain how it can be profitable to leave 10,000 seats empty.

A monopolist has an incentive to reduce quantity sold in order to increase price. Perhaps filling all 30,000 seats would require lowering the price to \$18, in which case it is clearly more profitable to sell only 20,000 tickets at the higher \$30 price.

b. Next week the Moons play the Tucson Turkeys, who have offered to buy an unlimited number of tickets at \$4 each, to be resold only in Tucson. How many tickets should be sold to Tucson to maximize profits — 10,000? More than 10,000? Fewer than 10,000?

There is some marginal cost associated with letting a fan into the stadium (say, the additional cleaning and security hours needed). In part a, the Moons set this marginal cost equal to their marginal revenue, and decided that profit was maximized by selling 20,000 tickets to Phoenix fans. Now, the Moons can sell an unlimited number of tickets to Tucson fans for \$4 each. This increases the marginal cost of letting a Phoenix fan into the stadium by \$4. In other words, there is now an additional \$4 opportunity cost associated with selling a ticket to a Phoenician relative to part a. Draw a picture. If the marginal cost has increased, clearly the profit-maximizing price in Phoenix has increased and the profit maximizing quantity decreased. In other words, the Moons should sell more than 10,000 tickets to Tucson and raise the price for a ticket in Phoenix to more than \$30.

c. Given your answer to b., what price should the Moons charge their own fans? \$4? \$5? More than \$5?

An excellent exercise is to paramaterize a linear demand curve (say with $mc = 0$) so that $p = \$30$ and $q = 20,000$ are the profit-maximizing price and quantity, and then solve explicitly for the profit-maximizing Phoenix price and quantity once the Tucson offer comes in.

Problem 3 An inventor has discovered a new method of producing a precious stone, using spring water found only in Venice, Italy and Danville, Kentucky. The process is patented and manufacturing plants are set up in both places. The product is sold only in Europe and the US. Trade laws are such that the price must be uniform within Europe and the US, but the European and American prices may differ. Transport costs are negligible, and there is no second-hand market in the stones because of the risk of forgeries. From the production and marketing data given below, determine the profit-maximizing production and sales plans. In particular, determine the output in Venice and Danville, sales in the US and Europe, quantity shipped from Europe to the US or vice versa, and prices in Europe and the US.

Demand: US, $p = 1500 - \frac{1}{2}Q$; Europe, $p = 1000 - Q$
Average cost: Danville, $AC = 150 + .375Q$; Venice, $AC = 100 + \frac{1}{2}Q$

A profit-maximizing monopolist will produce so that

$$MR_{US} = MR_E = MC_{US} = MC_E \tag{2}$$

Market clearing demands that

$$q^s_{US} + q^s_E = q^d_{US} + q^d_E \tag{3}$$

Solving this system yields $q^s_{US} = 600$ $q^s_E = 500$, $q^d_{US} = 900$, and $q^d_E = 200$. The price in the US will be $\$1,050$, and the price in Europe \$800. Europe will export 300 stones to the US.

Problem 4 MWG, problem 12.C.15

Observe here that the consumer indifferent between firms 1 and 2 is located at $\hat{z}$, where $p_1 + t\hat{z}^2 = p_2 + t(1-\hat{z})^2$, or $\hat{z} = \frac{t+p_2-p_1}{2t}$. As this is the same location of an indifferent customer as in the linear travel cost case studied in the text of MWG, each firm will have the same demand curve, and so will solve the same maximization problem, and so will set the same prices in a Nash equilibrium, $p_1 = p_2 = c + t$.

Problem 5 MWG, problem 12.C.16

You can solve this using a nearly identical technique to that of problem 4. There are $\frac{1}{J}$ customers between firms 1 and 2. Given prices p_1 and p_2, the customer indifferent between the 2 firms is located at distance x from firm 1, where

$$p_1 + tx^2 = p_2 + t(\frac{1}{J} - x)^2$$
$$\Rightarrow x = \frac{1}{2J} + \frac{J(p_2 - p_1)}{2t}$$

x is one of two components of firm 1's demand (he also gets customers on his other side, between firm 1 and firm J). In a symmetric equilibrium, in which all firms charge price p, firm i solves the following profit-maximization problem:

$$\max_{p_i}(p_i - c)(\frac{1}{J} + \frac{J(p - p_i)}{t}$$

which has FOC

$$p_i = \frac{t}{2J^2} + \frac{1}{2}(p - c)$$

In a symmetric equilibrium, $p_i = p$, or

$$p = \frac{t}{J^2} + c \tag{4}$$

while per-firm profits are given by $\frac{t}{J^3}$. Clearly, as $J \to \infty$, $p \to c$ and per firm profits are driven to zero.