Homework 7

due 4/29/2011

Problem 1 Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$2,000, and if an unqualified worker is assigned to the good job the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^2. The probability that an unqualified worker will have a test score less than t is t. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of \$4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $\$3,000$, for both A-workers and B-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

a. Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?

b. Now suppose that employers are subject to a rule that requires the proportion of A-workers assigned to the good job to be the same as the proportion of B-workers. Otherwise employers maximize expected profits. What is the effect of this rule?

Problem 2 Consider an economy in which there are equal numbers of men and women, and two kinds of jobs, good and bad. Some workers are qualified for the good job, and some are not. Employers believe that the proportion of men who are qualified is $\frac{2}{3}$ and the proportion of women who are qualified is $\frac{1}{3}$. If a qualified worker is assigned to the good job, the employer gains \$1,000, while if an unqualified worker is assigned to the good job, the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t. The probability that an unqualified worker will have a test score less than t is $t(2-t)$. Employers are subject to a rule that requires the proportion of men assigned to the good job to be the same as the proportion of women. Otherwise, employers maximize expected profits.

a. Find the profit-maximizing policy for an employer. Note that in this problem we take as given employer attitudes towards men and women; they do not need to be determined endogenously.

b. Test your policy as follows. If you are told that a worker has just barely passed the test (and you are not told whether the worker is male or female), what is the probability that the worker is qualified? Is it the case that such a worker is a fair bet from the employer's point of view? If not, should the policy be adjusted?

Problem 3 Suppose that business travelers have marginal willingness to pay $40 - q$ for a seat of quality $q \in [0, 40]$, meaning that their total willingness to pay for a seat of quality $\hat{q} \in [0, 40]$ is $\int_0^{\hat{q}} (40 - q)dq$ (assume that marginal willingness to pay is 0 for $q > 40$). Tourists have marginal willingness to pay of $30 - q$ for

$q \in [0, 30]$, meaning their total willingness to pay for a seat of quality $\hat{q} \in [0, 30]$ is $\int_0^{\hat{q}} (30 - q)dq$ (assume tourists have marginal willingness to pay of 0 for $q > 30$). Assume that 80 tourists and 20 business travelers typically fly a given route, and the the plane used on this route is more than big enough to hold all 100 travelers, so the airline never has to worry about a capacity constraint. However, the airline cannot tell which type a given traveler is, and so cannot condition price on group membership.

Suppose the airline is able to put two sections on the plane (i.e. 1st class and coach), each with its own quality level. Assume that the cost of setting quality level q in coach is $K_c * q$ and that the cost of setting quality q in 1st class is $K_{fc} * q$, for $K_{fc} \geq K_c$.

a. For parts a-d, set $K_{fc} = K_c = 0$. Suppose the airline sets $q = 30$ in coach and $q = 40$ in 1st class. Solve for the profit maximizing prices, taking these quality levels as given.

b. You are hired as a consultant to advise the airline on how it can increase profits. Explain why decreasing the quality in coach — and in turn decreasing the price — can increase the airline's profit, even if the number of passengers flying the route remains 100, with 80 tourists and 20 business travelers.

c. Solve for the profit-maximizing price and quality levels in both coach and business class.

d. Now suppose that the composition of travelers changes, so that fraction t of all travelers are business travelers, and fraction $1-t$ are tourists (the plane is still plenty big enough to hold all travelers, so constraints like there needing to be more seats in coach than there are passengers are not binding). Solve for the optimal price and quantity levels in coach and 1st class, as a function of t

e. Finally, suppose that $K_c = 1$ and $K_{fc} = \$K$. Suppose again that there are 80 tourists and 20 business travelers. Solve for the relationship between the price of coach and K, and give an intuitive explanation for why these two variables are related in this way.