Midterm exam

answers

Problem 1 (20 points (5, 5, 5, 5)) This problem asks you to produce examples of extensive form games with various characteristics. Be sure to give a complete description of the game (a picture is fine), as well as a clear explanation about why it has the required properties.

a. Give an example of a game that has a Nash equilibrium which is not subgame perfect.

See the example in Figure 9.C.5 in MWG.

b. Give an example of a game that has a subgame perfect equilibrium which is not perfect Bayesian.

See the example in Figure 9.C.1 in MWG.

c. Give an example of a game that has a perfect Bayesian equilibrium which is not subgame perfect.

See the example in Figure 9.C.5 in MWG.

d. Give an example of a game in which every Nash equilibrium is also a sequential equilibrium.

The prisoner's dilemma, written as a extensive form game. (Any extensive form game that has only one Nash equilibrium necessarily satisfies this criterion.)

Problem 2 (36 points (14, 14, 8)) Consider the normal form game G below

a. Compute all Nash equilibria of G.

First, note that B is strictly dominated for 1 by $\frac{7}{10}A + \frac{3}{10}B$, and so is not played in any Nash equilibrium. Go through an exhaustive list of supports. Suppose 1 plays A. Then 2 prefers b, which means 1 would prefer C, and so there is no equilibrium in which 1 plays A. Now, suppose 1 plays C. Then 2 prefers a, but then 1 prefers A, and so there is no equilibrium in which 1 plays C. Therefore, 1 mixes between A and C in any Nash equilibrium. 1 is willing to do this only if $a = \frac{4}{5}c$. 2 is indifferent between a and c iff 1 plays $\frac{1}{4}A + \frac{3}{4}C$. Therefore, the game's lone equilibrium is at $(\frac{1}{4}A + \frac{3}{4}C, \frac{4}{9}(1-x), \frac{5}{9}(1-x), x)$, for all $x \in [0, 1]$.

b. Sketch the set of payoffs which are obtainable in a Nash equilibrium of the infinitely repeated game $G^\infty(\delta)$ if δ is close enough to 1.

From inspection of each player's best response correspondences, 1's minmax payoff is 2 (ensured if 2 plays c), and player 2's minmax payoff is $\frac{13}{4}$ (ensured if 1 plays $\frac{1}{4}B + \frac{3}{4}C$). The set of payoffs supportable in a Nash equilibrium if δ is high enough is then $F \bigcap IR$, with IR determined by all points dominating each player's minmax payoff, and F equal to the convex hull of the payoff vectors from all 9 pure strategy profiles.

c. Suppose that σ is a strategy profile for $G^\infty(\delta)$ which yields payoffs outside of the closure of the set you sketched in part b. (this means that the payoffs are neither in the set, nor on the boundary of the set). Prove that σ is not a Nash equilibrium of $G^\infty(\delta)$.

If σ yields payoffs outside of the closure of the set $F \bigcap IR$, then at least one player gets strictly less than his minmax value in the repeated game. Suppose WLOG that player 1 is such a player, obtaining a payoff of less than his minmax payoff of 2 under σ. If, however, instead of playing σ_1 in response to σ_2, player 1 deviated to, for example, always playing strategy C no matter what player 2 plays, he then guarantees himself a payoff of at least 2, his minmax payoff. Since this is clearly an optimal deviation for player 1, σ cannot be a Nash equilibrium strategy.

Problem 3 (15 points) For this question, let G be a two-player zero-sum game (hint: while G is an arbitrary zero-sum game, it may be helpful to think of a particular, simple zero-sum game in answering this question, such as matching pennies).

a. Suppose that $\sigma_1 \in \Delta S_1$ is a rationalizable strategy for player 1. Must σ_1 be a maxmin strategy for player 1? Provide a proof or counterexample.

This is false. Consider the matching pennies game below:

1 \ 2	h	t
H	$-1, 1$	$1, -1$
T	$1, -1$	$-1, 1$

All elements of $\Delta\{H, T\}$ are rationalizable for player 1. Yet 1's unique maxmin strategy is $\frac{1}{2}H + \frac{1}{2}T$, giving him a maxmin payoff of 0.

b. Suppose that $\sigma_1 \in \Delta S_1$ is a maxmin strategy for player 1. Must σ_1 be a rationalizable strategy for player 1? Provide a proof or counterexample.

This is true. If $\overline{\sigma_1}$ is a maxmin strategy for player 1, then it is a best response to some minmax strategy for player 2, say $\underline{\sigma_2}$, and $(\overline{\sigma_1}, \underline{\sigma_2})$ is a Nash equilibrium, since G is a zero sum game. Since $\overline{\sigma_1}$ is a Nash equilibrium strategy, it is necessarily rationalizable.

Problem 4 (14 points) Suppose that players 1 and 2 play the game below, and that it is common knowledge between them that both of them are rational. If we make no other assumptions about the players' knowledge, what is our best prediction about how they will play the game?

The best prediction we can make is that they will play rationalizable strategies. Since this is a 2-player game, the rationalizable strategies are the same as those that survive iterated removal of strictly dominated strategies. From inspection, c is strictly dominated by a for player 2, and so is never a best response. If 1 knows that 2 will never play c, then A is not a best response to any of 2's remaining strategies. Then, if 2 knows that 1 will never play A, a is strictly dominated by $.6d + .4b$, and so a rational 2 will never play a. No further pure strategies can be eliminated.

Draw best response correspondences for each player. For 1, B alone is a BR to every one of player 2's remaining strategies, except for b, for which both B and C are best responses. Therefor, B, C, and all mixtures of B and C are rationalizable for player 1. For 2, b is a best response to any player 1 strategy which puts weight of $\frac{2}{5}$ of more on C, d is a best response to any strategy which puts weight of $\frac{2}{5}$ or less on C, and e is a best response to 1 playing a pure strategy of B. Therefore, b, d, e, all mixtures of d and e, and all mixtures of b and d are rationalizble for player 2 (but *not* any strategy which place positive weight on both b and e).

Problem 5 (15 points (10, 5)) Consider the 3-player game between two entrants ($E1$ and $E2$) and an incumbent (I) in figure 2. Entrant $E1$ will either enter on his own or as part of a joint venture with entrant $E2$. If enrty occurs, firm I chooses between fighting entry with a price war and acquiescing to normal competition. Firm I cannot observe whether or not entry was the result of a joint venture.

a. Find all perfect Bayesian equilibria of this game.

This is example 9.C.2 in MWG.

b. Explain why the perfect Bayesian refinement is needed in this game. Why is NE or SPE unsatisfactory?

The perfect Bayesian refinement codifies the principle of *backward induction*, that off-equilibrium path behavior should be credible. Note that ((Out, Out if E2 declines) Decline, fight) is a Nash equilibrium in which entry is deterred by not only the threat of a price war, but also by $E2$ declining any offer of a joint venture. As declining is a strictly dominated strategy for $E2$, this is strange behavior. It seems unlikely that were $E2$'s information set to be reached, that she would actually play "Decline". Therefore, requiring her to behave sensibly, even at an unreached information set, seems reasonable. Nash equilibrium (or even SPE) does not require this, and so the PBE refinement, which says that players cannot play strictly dominated strategies at unreached information sets, is needed.