Expected utility

Let Z be some set of prizes/outcomes (for example, Z could include three dollar amounts, \$100, \$10, \$0, and you will get one).

Let $\succsim$ denote **preferences** over Z. As it turns out, we need preferences not only over Z, but over ΔZ, the set of **compound lotteries** from Z (i.e. is a 95% chance you get \$0 and 5% chance of \$100 better or worse than a 100% chance of \$10?).

As usual, $p \succ q$ means $p \succsim q$ but not $q \succsim p$.

Axioms on preferences

(VNM1) (weak order): $\succsim$ is complete and transitive

(VNM2) (continuity): for $p, q, r \in Z$ such that $p \succ q \succ r$, there are numbers δ and ϵ, both in $(0, 1)$ satisfying

$$(1-\delta)p + \delta r \succ q \succ (1-\epsilon)r + \epsilon p$$

(VNM3) (independence of irrelevant alternatives): For all $p, q, r \in Z$, and for all $\alpha \in [0, 1]$,

$$p \succsim q \iff \alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$$

The following two statements are equivalent:

1. $\succsim$ satisfies VNM1-VNM3

2. There exists a function $u : Z \to \mathbb{R}$ such that
$$p \succsim q \iff \sum_{z \in Z} u(z)p(z) \geq \sum_{z \in Z} u(z)q(z)$$