

### Example: bargaining game

Consider the following game played between two unions and a firm. First, union 1 decides either to make a concession to the firm or not to make a concession. Union 2 observes union 1's action, and then decides whether or not to make a concession to the firm. The firm then chooses between production plan  $a$  and production plan  $b$ . The firm has two information sets: it either observes that both firms have made concessions, or it observes that at least one firm did not make a concession.

Each union obtains \$4 if the firm chooses plan  $a$  and \$0 if the firm chooses plan  $b$ . In addition, each union loses \$2 for making a concession. If the firm chooses plan  $a$ , it obtains \$2 for each union that makes a concession. If the firm chooses plan  $b$ , it obtains \$1 for certain, and receives an additional \$1 if both unions make concessions. Utilities are linear in dollars.

a. Draw an extensive form representation of this game, and find all of its sequential equilibria.

As shown in class, the sequential equilibria of this game are:

$$(C, (c, \hat{d}), (A, b)) \text{ with } \mu_3(z) \geq \frac{1}{2}$$

$$(C, (c, \hat{d}), (A, \sigma_3(b) \geq \frac{1}{2})) \text{ with } \mu_3(z) = \frac{1}{2}$$

These equilibria differ only in the probability 3 assigns to  $b$  at his unreached information set; I would say the most accurate way to think of this game is that there is one equilibrium, but that 3 has some freedom to change his strategy. All equilibria yield payoffs  $(2, 2, 4)$ .

b. Now suppose that union 2 cannot observe union 1's decision. Draw an appropriate extensive form for this new game, and compute all of its sequential equilibria. If you would like additional practice looking for sequential equilibria, this would be an excellent problem to try, and I'd be happy to help you along. The answer is that there is a unique sequential equilibrium at  $(D, d, (A, b))$  with  $\mu_3(z) = 1$  and 2 believing she is at her rightmost node with probability 1. Note that 2 now only has one behavior strategy, as her 2 decision nodes are in the same information set in this version of the game.

c. The games in parts a-b each have a unique sequential equilibrium outcome, but the choices made on these equilibrium paths are quite different. Explain in words why the choices made on the equilibrium path in a cannot be made on the equilibrium path in b, and vice-versa. Evidently, the differences here must hinge on whether or not player 2 can observe a deviation by player 1.